Varieties of Quantity Estimation in Children

Article *in* Developmental Psychology · June 2015

DOI: 10.1037/a0039183 · Source: PubMed

4 authors:

Francesco Sella
University of Oxford

Ilaria Berteletti
Gallaudet University

Daniela Lucangeli
University of Padova

Marco Zorzi
University of Padova

Developmental Psychology
2015, Vol. 51, No. 6, 758–770

0012-1649/15/$12.00 http://dx.doi.org/10.1037/a0039183

Varieties of Quantity Estimation in Children

Francesco Sella
University of Padova

Ilaria Berteletti
University of Illinois, Urbana Champaign

Daniela Lucangeli
University of Padova

Marco Zorzi
University of Padova and IRCCS San Camillo Hospital, Venice-Lido, Italy

In the number-to-position task, with increasing age and numerical expertise, children's pattern of estimates shifts from a biased (nonlinear) to a formal (linear) mapping. This widely replicated finding concerns symbolic numbers, whereas less is known about other types of quantity estimation. In Experiment 1, Preschool, Grade 1, and Grade 3 children were asked to map continuous quantities, discrete nonsymbolic quantities (numerosities), and symbolic (Arabic) numbers onto a visual line. Numerical quantity was matched for the symbolic and discrete nonsymbolic conditions, whereas cumulative surface area was matched for the continuous and discrete quantity conditions. Crucially, in the discrete condition children's estimation could rely either on the cumulative area or numerosity. All children showed a linear mapping for continuous quantities, whereas a developmental shift from a logarithmic to a linear mapping was observed for both nonsymbolic and symbolic numerical quantities. Analyses on individual estimates suggested the presence of two distinct strategies in estimating discrete nonsymbolic quantities: one based on numerosity and the other based on spatial extent. In Experiment 2, a non-spatial continuous quantity (shades of gray) and new discrete nonsymbolic conditions were added to the set used in Experiment 1. Results confirmed the linear patterns for the continuous tasks, as well as the presence of a subset of children relying on numerosity for the discrete nonsymbolic numerosity conditions despite the availability of continuous visual cues. Overall, our findings demonstrate that estimation of numerical and non-numerical quantities is based on different processing strategies and follow different developmental trajectories.

Keywords: number line estimation, numerical cognition, children, numerical estimation, nonnumerical estimation

A growing number of studies have demonstrated that humans and many animal species can represent and operate on approximate numerical quantities (Agrillo, Dadda, Serena, & Bisazza, 2009; Cantlon & Brannon, 2006). However, numerate humans are also able to represent numerical quantities in an exact way as a consequence of learning numerical symbols (Verguts, Fias, & Stevens, 2005; Zorzi & Butterworth, 1999; Zorzi, Stoianov, & Umiltà, 2005).

A window into the transition between approximate and exact estimation of symbolic numbers is offered by a numerical estimation task that requires individuals to map a given numerical value onto a visual line, widely known as the number-to-position (NP) task (Siegler & Opfer, 2003). In a seminal study, Siegler and Opfer (2003) asked children to indicate where a given number (e.g., 25) should be placed onto a black horizontal line with the left and right ends labeled as 0 and 100 (or 1000), respectively. Younger children displayed a pattern of estimates characterized by overestimation of small numbers and underestimation of larger numbers. With increasing age and education (in particular, familiarity with the tested numerical range), children shift from this biased estimates to a formal and linear estimation that entails the accurate placement of numbers (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Booth, 2004; Siegler & Opfer, 2003). Siegler & Opfer (2003) originally argued that the biased pattern of estimates was well described by a logarithmic function, which is consistent with the widely accepted notion of logarithmically compressed representation of numerical magnitude (i.e., the approximate number system; ANS; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; Stoianov & Zorzi, 2012). Accordingly, the typical pattern of biased estimates in the NP task entails a greater distance between small numbers compared with larger numbers, thereby suggesting that young children used the logarithmic and more biased representation (i.e., the ANS) to accomplish the task.

Although the idea of a developmental shift from a biased to a linear mapping is widely accepted (Siegler, Thompson, &

Francesco Sella, Department of General Psychology, University of Padova; Ilaria Berteletti, Department of Psychology, University of Illinois at Urbana–Champaign; Daniela Lucangeli, Department of Developmental Psychology and Socialization, University of Padova; Marco Zorzi, Department of General Psychology and Center for Cognitive Neuroscience, University of Padova, and IRCCS San Camillo Hospital, Venice-Lido, Italy.

Francesco Sella is now at the Department of Experimental Psychology, Oxford University.

This study was partially supported by University of Padova Strategic Grant "NEURAT" to Marco Zorzi. We thank all children and their teachers and parents for participating in the present study, as well as undergraduate research assistants for their help in collecting data.

Correspondence concerning this article should be addressed to Marco Zorzi, Dipartimento di Psicologia Generale, via Venezia 8, 35131 Padova, Italy. E-mail: marco.zorzi@unipd.it

Opfer, 2009), there is a lively debate on the nature of the biased pattern of estimates, both from a statistical (i.e., best fitting model) and a theoretical perspective (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Cohen & Sarnecka, 2014; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009). For example, the familiarity model (Ebersbach et al., 2008; Moeller et al., 2009) assumes that the compressed pattern of estimates is best fit by two separated straight lines (i.e., a bilinear function), fitting familiar single-digit numbers separately from less-familiar two-digit numbers. The proportional model, instead, considers the pattern of estimates in the NP task as evidence of a proportional judgment process (Barth & Paladino, 2011, but see Opfer, Siegler, & Young, 2011 for a convincing confutation; Hollands & Dyre, 2000; Spence, 1990). Irrespective of the outcome of the above-mentioned debate, an open issue is whether the developmental shift observed for the NP task is tied to the type of items to be estimated. Although the same developmental shift from biased to linear estimation was observed in many different numerical tasks with different scales (Siegler et al., 2009), as well as for nonnumerical ordered sequences (Berteletti, Lucangeli, & Zorzi, 2012), there is sparse evidence on the estimation of nonsymbolic quantities. Booth and Siegler (2006) found the logarithmic to linear shift also when children had to transcode Arabic digits into continuous (i.e., line length) or discrete (e.g., dots) quantities.

Here we question whether the estimation of nonsymbolic quantities, discrete and continuous, would show a similar or a different estimation pattern compared with the one showed by children when estimating symbolic numerical quantities of equal value. By comparing estimation for different types of quantities, it will be possible to determine whether the pattern observed in the symbolic NP task is the consequence of poor knowledge of the elements of the specified numerical interval, or whether children rely on common representations for different types of quantities (i.e., symbolic, discrete or continuous). In Experiment 1, we compared the symbolic NP task with two novel tasks that are directly matched in terms of quantities: a nonsymbolic discrete task (i.e., sets of objects) and a nonsymbolic continuous task (i.e., spatial extent). In Experiment 2, we tested the same conditions of Experiment 1 (including some variants) as well as a new line mapping task in which the quantity was continuous and nonspatial (i.e., shades of gray). The cross-sectional design allowed us to assess whether the well-known developmental trajectory in the symbolic NP task can be observed for the other types of quantity estimations. We would like to highlight that the ongoing debate about which model best captures the developmental change in the pattern of estimates in the NP-task is orthogonal to the aims of the present study. For this reason, we simply refer to the classic distinction between logarithmic and linear positioning without assuming that the selected model is a faithful index of the underlying representation (Karolis, Iuculano, & Butterworth, 2011; Moeller et al., 2009). However, the developmental change should only occur for numerical quantities if it is related to an increased mastery of the numerical values and the principles that underlie the numerical system.

Experiment 1

We directly compared three estimation tasks that differed in the format of the quantities to be mapped onto the visual line: symbolic (i.e., the classic NP-task), discrete (i.e., numerosity), and continuous (i.e., spatial extent; see Figure 1). In the two latter versions, children had to position nonsymbolic quantities onto lines that were bounded at the left end by an empty square—corresponding to zero—and at the right end by a square that was either filled with 100 objects or completely black—corresponding to the maximum possible quantity. In the continuous quantity estimation, the items to be placed on the line were represented by a growing black rectangle progressively filling a box; in the discrete condition, the items were represented by sets of equal size squares progressively filling a box. Crucially, the discrete quantity could be processed either as numerosity or as continuous quantity, because the numerosity of each set was perfectly correlated to its cumulative surface area (i.e., the sum of the area of all squares in the discrete condition matched the area occupied by the black rectangle in the continuous condition). Moreover, the quantities were exactly the same across the three conditions to make a direct comparison of the patterns of estimates (e.g., 25% of fullness in the continuous condition, 25 squares in the discrete condition, and the number "25" in the symbolic condition).

In the continuous condition, the quantity (black area) had to be mapped onto another continuous quantity (length of the line). Thus, we predicted that the estimates would be fairly linear even for young children because the transformation takes place within

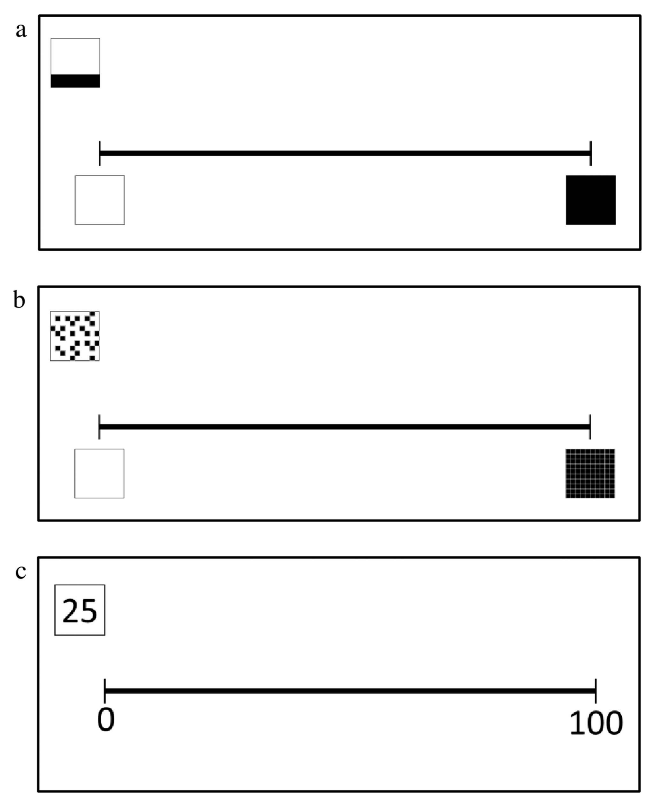

Figure 1. Example of three trials with (a) continuous, (b) discrete, and (c) symbolic representation of the same quantity (i.e., 25).

the visuospatial domain (similarly to a simple proportional judgment; Spence, 1990). In the symbolic condition, we expected to observe the widely observed logarithmic-like pattern with a shift to linearity as a function of age (Barth & Paladino, 2011; Berteletti et al., 2010; Booth & Siegler, 2006; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Moeller et al., 2009; Siegler & Booth, 2004; Siegler & Opfer, 2003). The discrete condition could yield variable mappings depending on how the discrete quantities are processed. If children use the continuous visual cues (i.e., cumulative surface area; Clearfield & Mix, 1999, 2001; Mix, Huttenlocher, & Levine, 2002) as input to the estimation process, the type of mapping should mirror the one observed in the continuous condition. In contrast, if children automatically encode numerosities (Cantlon, Safford, & Brannon, 2010; Cordes & Brannon, 2008, 2009; Stoianov & Zorzi, 2012), we would expect children to display a pattern of estimates with a marked overestimation of small numerosities (Dehaene, Izard, Spelke, & Pica, 2008), more similar to the symbolic condition (Berteletti et al., 2010; Siegler & Opfer, 2003). Across grades, the estimation of discrete quantities might become more accurate, possibly reaching a linear mapping as recently observed in adults (Anobile, Cicchini, & Burr, 2012, but see Dehaene et al., 2008; Núñez, Doan, & Nikoulina, 2011). Indeed, Anobile et al. (2012) using a bounded number line task in which participants mapped numerosities identical to those used in the classic study of Siegler and Opfer (2003) but in the discrete format (i.e., dots), showed that educated adults deploy an accurate linear mapping in this task, unless attentional resources were diverted by a dual task manipulation. The direct comparison between different formats can also shed light on the type of processes used by children to solve the NP task. Indeed, if children rely on the same mapping mechanism for estimating nonsymbolic quantities, we should observe the same pattern of estimates as the one observed when symbolic quantities are positioned on the line. This would support the assumption that the pattern observed in the symbolic task is not the consequence of poor knowledge of the elements to be positioned or the properties of the interval. By testing children from preschool to Grade 3, we investigated whether the estimation of nonsymbolic quantities also changes with development.

Method

Participants. Two hundred and three children from preschool to Grade 3 were recruited from schools located in northern Italy. All children spoke Italian as a first language and they were mostly of middle socioeconomic status. There were 40 preschoolers (17 boys; age range = 5–6), 68 from Grade 1 (30 boys; age range 6–7) and 95 from Grade 3 (44 boys; age range = 7–8). We selected this particular age range to maximize the possibility of observing a developmental change in the NP task (Siegler & Opfer, 2003). In particular, a recent study on Italian children (Berteletti et al., 2012) reported a logarithmic pattern for preschoolers, a linear pattern for third graders, and an intermediate pattern for first graders. A replication of the typical developmental pattern for the NP task (i.e., symbolic condition) provides an important baseline for highlighting similarities and differences between the different types of quantity estimation.

Procedure. Children were met individually, in a quiet room, and completed the three paper-and-pencil estimation tasks. The order in which the children completed the experimental tasks was determined randomly. The stimuli in each task were also ordered randomly. The estimation tasks were presented as games, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop at any time.

Tasks. The nonsymbolic estimation tasks are adaptations from the NP task of Siegler and Opfer (2003). For all three conditions, a 20-cm black line was presented in the center of a half A4 landscape white sheet (see Figure 1). In the symbolic condition, the left end was labeled 0 and the right end was labeled 100. Children were required to estimate the position of 10 numbers (i.e., 2, 3, 4, 6, 18, 25, 42, 67, 71, 86; adapted from Siegler & Opfer, 2003), making a pen mark on the line. For each trial, the number to be positioned was presented inside a box in the upper left corner of the sheet. For the continuous condition, an empty box (2 × 2 cm) was placed just below the left end of the line, whereas a full black box was placed just below the right end. Children were told that the black box was a box full of liquid (e.g., juice) and the other one was empty and the horizontal line meant the level of fullness. The quantity to be positioned was represented by a partially filled box (i.e., 2%, 3%, 4%, 6%, 18%, 25%, 42%, 67%, 71%, or 86% full) placed in the upper left corner. For the discrete condition, the same empty box was placed just below the left end, and a box filled with 100 small black squares (0.2 × 0.2 cm) was placed just below the right-end of the line. The quantity to be positioned was represented by a box filled with a variable amount of randomly spread small squares (i.e., 2, 3, 4, 6, 18, 25, 42, 67, 71, or 86 squares). Children were told that the squares were chocolate pieces and the line went from an empty box to a full box of chocolate pieces. Children were not allowed to count the squares. Instructions were similar for the three estimation tasks except for specific changes for each type of stimuli. Symbolic condition instructions were

> *We will now play a game with number lines. In this page there is a line that goes from 0 to 100. In the upper left box there is a number that I want you to place on the line making a mark using your pencil.*

While pointing to the relevant elements on the sheet, the experimenter went on with the question, *If 0 is here and 100 is here, where would you place 25?* Discrete/continuous condition instructions were

> *We will now play a game. In this page there is a line that goes from an empty box of chocolate/juice to a full box of chocolate/juice. In the upper left box there is a quantity of chocolates/juice that I want you to place on the line making a mark using your pencil.*

While pointing to the relevant elements on the sheet, the experimenter went on with the question, *If the empty box is here and the full box is here, where would you place this quantity of chocolates/juice?* To verify whether children had understood the question and were aware of the interval size, they were asked to place 0 (empty box) and 100 (full box) on the line. Only on these two practice trials the experimenter gave feedback for wrong responses by saying: "This line goes from 0 (empty box) to 100 (full box), if I want to place 0/100 (empty box/full box), this is the right place." After the two examples, the task started and no other feedback was given.

Results

In case of nonsphericity for the analyses of variance (ANOVAs), we use the Greenhouse–Geisser correction for p values (Field, Miles, & Field, 2012). We also report the generalized eta-square as a measure of effect size (Bakeman, 2005; Field et al., 2012). The p values for planned comparisons are corrected using the Bonferroni formula. All the analyses were conducted in the R environment (R Development Core Team, 2013) using ez package (Lawrence, 2013) and ggplot2 package (Wickham, 2009).

Group analysis. Estimation accuracy was assessed using the percentage of absolute error (PAE) for each participant and condition. This was calculated as follows: PAE = (|Estimate − Target number or quantity|/scale of estimation) × 100. A mixed ANOVA was computed with grade as between-subjects factor (preschool, Grade 1, and Grade 3) and condition as within-subject factor (continuous, symbolic, and discrete). Mean PAEs, from preschool to Grade 3, were 19%, 15%, and 11% in the continuous condition; 21%, 20%, and 13% in the discrete condition; and 24%, 18%, and 9% in the symbolic condition (see Figure 2). The main effect of condition $F(2, 400) = 6.84$, mean square error (MSE) = 56.19, $p = .001$, $\eta^2_g = 0.01$ and the main effect of Grade, $F(2, 200) = 29.95$, $MSE = 175.28$, $p < .001$, $\eta^2_g = 0.15$, were significant. Given that the interaction was also significant, $F(4, 400) = 6.55$, $MSE = 56.19$, $p < .001$, $\eta^2_g = 0.02$, we performed separate repeated-measures ANOVAs for each grade with condition as the within-subject factor. Condition was significant for the three separate ANOVAs showing a difference in precision of estimation as a function of condition, preschool: $F(2, 78) = 5.9$, $MSE = 50.21$, $p = .004$, $\eta^2_g = 0.04$; Grade 1: $F(2, 134) = 5.91$, $MSE = 80.16$, $p = .003$, $\eta^2_g = 0.04$; Grade 3: $F(2, 188) = 8.52$, $MSE = 41.59$, $p < .001$, $\eta^2_g = 0.03$. In preschool children, planned

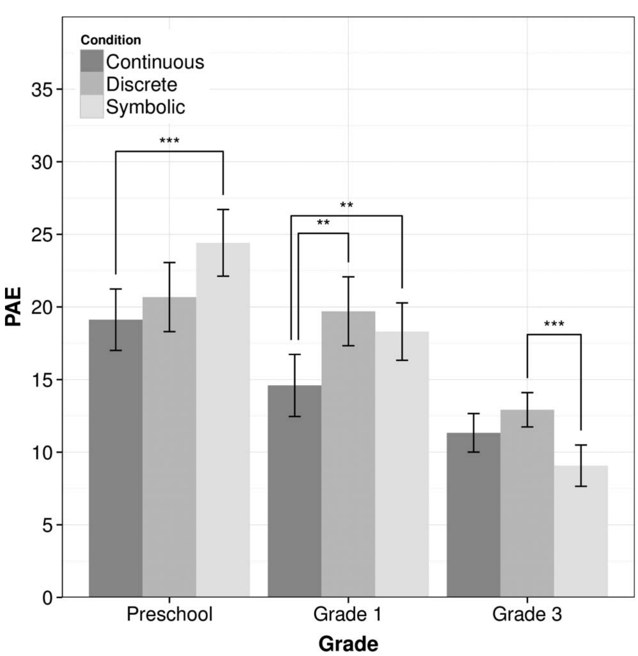

Figure 2. Percentage of absolute error (PAE) as a function of grade and condition. Bars represent within-subjects 95% confidence interval (Morey, 2008). All ps are Bonferroni-corrected. ** $p < .01$. *** $p < .001$.

t test comparisons (we reported Bonferroni-adjusted ps for 3 comparisons) revealed that preschool children were more accurate in the continuous condition compared with the symbolic condition, $t(39) = 3.53$, $p = .001$; in Grade 1, pupils were more precise in the continuous condition compared with both the discrete and the symbolic condition, $t(67) = 3.08$, $p = .003$; $t(67 = 2.7$, $p = .009$, respectively); finally, Grade 3 pupils had a better estimation accuracy in the symbolic condition compared with the discrete condition, $t(94) = 4.14$, $p < .001$.

To understand the pattern of estimates for each condition, we first fit group medians and then the individual data (Siegler & Opfer, 2003). Because we do not attempt to solve the debate on which model best describes performance, we use the models that have traditionally been used to observe the developmental improvement of estimation, namely, the logarithmic and the linear function. Group median estimates and the corresponding best linear or logarithmic fit are reported in Figure 3. To asses which model describes performances more accurately in each grade, the difference between linear and logarithmic models was tested with a paired-sample t test on absolute distances between children's median estimate for each number and the predicted values according to the linear and the logarithmic model. If the t test was significant, the best fitting model was attributed to the group. In the continuous condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for all groups, preschool: $t(9) = 3.5$, $p = .007$, linear $R^2 = 96\%$ vs. log $R^2 = 73\%$; Grade 1: $t(9) = 4.23$, $p = .002$, linear $R^2 = 98\%$ versus log $R^2 = 75\%$; Grade 3: $t(9) = 4.22$, $p = .002$, linear $R^2 = 98\%$ versus log $R^2 = 75\%$). In the discrete condition, for preschool and Grade 1, the difference between the two models did not reach significance, indicating an intermediate stage, preschool: $t(9) = 1.64$, $p = .135$, linear $R2 = 97\%$ versus log $R^2 = 91\%$; Grade 1: $t(9) = 1.61$, $p = .142$, linear $R^2 = 93\%$ versus log $R^2 = 98\%$. For Grade 3 children, the linear model showed the best fit, $t(9) = 2.52$, $p = .033$, linear $R^2 = 98\%$ versus log $R^2 = 92\%$. Finally, in the symbolic condition, the logarithmic model had the highest R^2 for both preschool and Grade 1 and significantly differed from the linear model, preschool: $t(9) = 3.92$, $p = .003$, linear $R^2 = 78\%$ versus log $R^2 = 98\%$; Grade 1: $t(9) = 2.76$, $p = .022$, linear $R^2 = 88\%$ versus log $R^2 = 99\%$). For Grade 3 children, the linear fit was significantly better, $t(9) = 2.35$, $p = .043$, linear $R^2 = 98\%$ versus log $R^2 = 90\%$.

Individual analysis. We fit the linear and the logarithmic model on individual estimates for each condition. Whenever both models did not reach significance, the child was classified as not able to complete the task properly ("none" category; Berteletti et al., 2010). When at least one model was significant, the highest R^2 determined the type of mapping displayed by the child. Indeed, when estimates are almost linear, the logarithmic model also fits very well the data. Table 1 shows the percentages of children with each type of mapping for each condition. Inspection of the table reveals that, in the continuous condition, for all the three age groups a high percentage of children displayed a linear mapping whereas only few of them showed a logarithmic mapping. In the discrete condition, there is a slightly higher percentage of children displaying a logarithmic mapping for preschoolers and first graders, whereas for the third graders the pattern in reversed. In the symbolic condition, there is a high percentage of preschoolers and

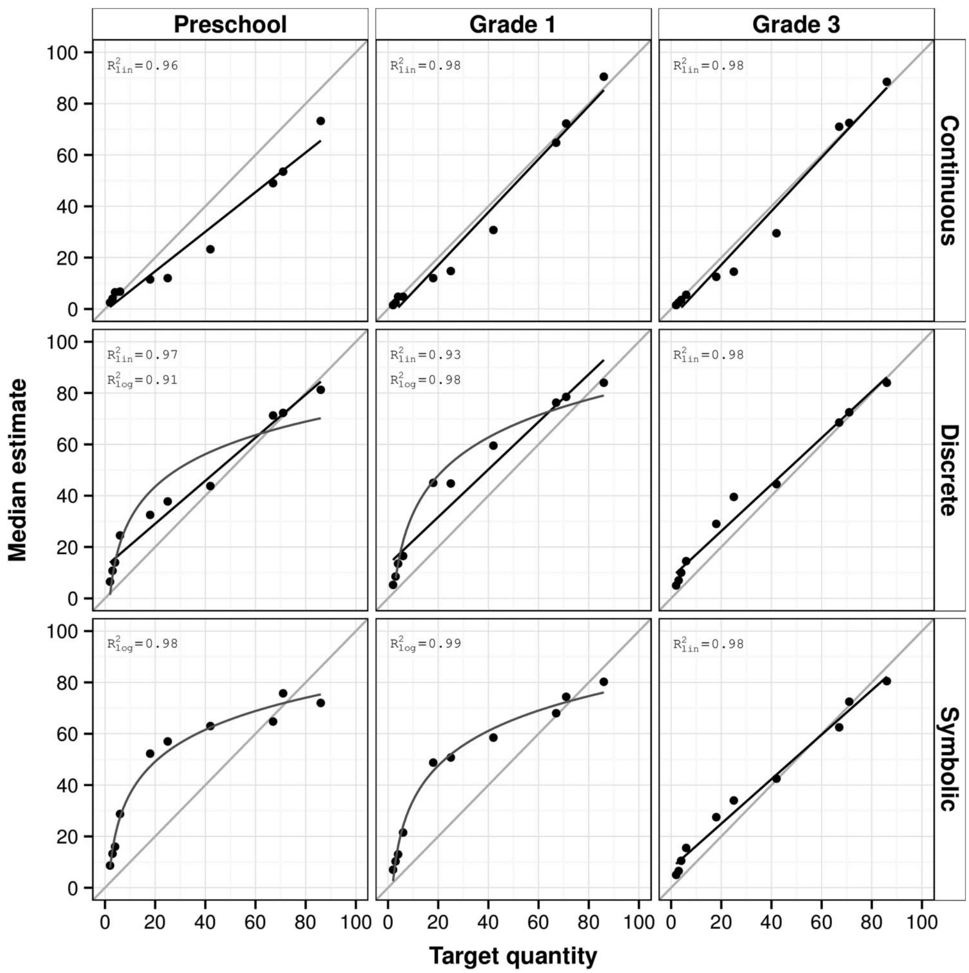

Figure 3. Median estimates and best fitting models as a function of grade for continuous, discrete, and symbolic conditions.

third graders displaying a logarithmic mapping whereas the majority of third graders are classified as linear.

Overall, at the group level, the symbolic condition showed the previously described developmental pattern with a progressive shift from logarithmic to linear positioning. For the continuous condition, children as young as 5 years old were already able to linearly map continuous quantities on to the line. For the discrete condition, both the linear and logarithmic models fit group medians for preschool and Grade 1 children. Moreover, at individual subject level, we observed that for both the symbolic and the discrete conditions, a large percentage of children were classified as positioning items following a logarithmic distribution whereas only a very small number of kids were doing so for the continuous condition (i.e., approximately 6%). In the symbolic condition, fewer children in Grade 3 showed a logarithmic pattern of responding than those in preschool or Grade 1, whereas in the discrete condition, the patterns of responding were similar across grades. We selected only those children who performed a linear mapping in the continuous condition and we questioned whether their mapping remained linear in the discrete condition. These children achieved a linear mapping in the continuous condition translating the physical dimension of the target quantity (i.e., fullness of the box) into the corresponding position on the line. The central question is whether they keep on relying on physical dimension of the stimuli also for the discrete condition. Of the 167 children with a linear mapping in the continuous condition, 7% were classified as none, 42% as logarithmic, and 51% as linear in the discrete condition. The same analysis was run separately for each grade and among children with linear mapping in the continuous condition, a similar percentage of children across grades displayed a biased (logarithmic) mapping in the discrete condition, preschool: 17% none, 38% logarithmic, 46% linear; Grade 1: 12% none, 50% logarithmic, 38% linear; Grade 3: 1% none, 38% logarithmic, 61% linear. Within each group, a considerable amount of children showed a bias in estimation similar to the one observed in the symbolic task: overestimating small quantities and underestimating larger ones.

To further investigate the relation between the type of mapping in the discrete and the symbolic estimation tasks, we calculated the percentage of children, separately for each grade (the "none" category was excluded), deploying either logarithmic or linear mapping in the symbolic condition as a function of the type of

Table 1
Types of Mapping in the Three Age Groups for Continuous, Discrete, and Symbolic Conditions

	Type of mapping		
Condition and grade	None	Logarithmic	Linear
Continuous			
Preschool	27.5	12.5	60.0
Grade 1	10.3	4.4	85.3
Grade 3	5.3	5.3	89.5
Discrete			
Preschool	25.0	40.0	35.0
Grade 1	13.2	50.0	36.8
Grade 3	2.1	38.9	58.9
Symbolic			
Preschool	22.5	70.0	7.5
Grade 1	5.9	79.4	14.7
Grade 3	1.1	26.3	72.6

Note. Cell values represent percentages of children with row sums equal to 100%.

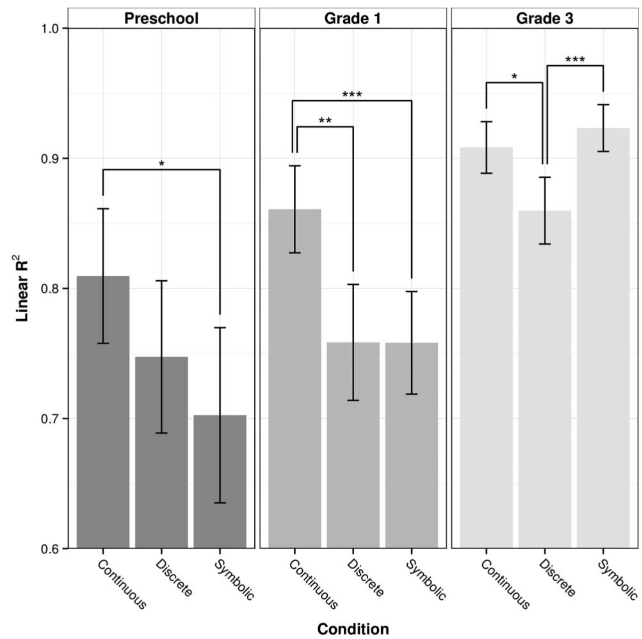

Figure 4. Linear R^2 as a function of grade for continuous, discrete, and symbolic conditions. All *p*s are Bonferroni-corrected. $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

mapping observed in the discrete condition. As can be noted from Table 2, approximately 60% of children across the three age groups displayed a concordance in mappings between discrete and symbolic conditions. For preschool and Grade 1 children, this is shown by the high percentage of children in the logarithmic–logarithmic intersection, whereas for the Grade 3 children the pattern is reversed, with a high percentage of children in the linear–linear intersection.

In summary, individual analyses suggest that children used two distinct mapping patterns when estimating discrete quantities. Approximately half of the children who were linear in the continuous condition deployed the same pattern when processing stimuli in the discrete condition. The other half showed a pattern of estimates that mirrored the symbolic condition, suggesting that these children might have focused on numerosity rather than spatial extent.

***R^2* of linear fit.** To further investigate the progression toward a linear mapping with grade in the three conditions, we used the linear R^2 as index of linearity for all the children who displayed logarithmic or linear mappings in all conditions (see Figure 4). There were 23 preschoolers, 51 first graders, and 89 third graders. We analyzed the linear R^2 in a mixed ANOVA with grade as between-subjects factor (preschool, Grade 1, and Grade 3) and condition as within-subject factor (continuous, symbolic, and discrete). The main effect of grade was significant, $F(2, 160) = 34.69$, $MSE = 0.02$, $p < .001$, $\eta_g^2 = 0.17$, as well as the main effect of condition, $F(2, 320) = 13$, $MSE = 0.01$, $p < .001$, $\eta_g^2 = 0.04$. The interaction Grade × Condition also reached significance, $F(4, 320) = 5.44$, $MSE = 0.01$, $p < .001$, $\eta_g^2 = 0.04$. We performed repeated-measures ANOVAs for each grade separately with condition as within-subject factor. The effect of condition was significant for all three age groups, preschool: $F(2, 44) = 3.5$, $MSE = 0.02$, $p = .039$, $\eta_g^2 = 0.08$; Grade 1: $F(2, 100) = 9.09$, $MSE = 0.02$, $p < .001$, $\eta_g^2 = 0.09$; Grade 3: $F(2, 176) = 9.5$, $MSE = 0.01$, $p_{[GG]} < .001$, $\eta_g^2 = 0.06$. Planned *t* test (we reported Bonferroni-adjusted *p*s for 3 comparisons) revealed that the linear fit was higher in the continuous condition compared with the symbolic condition for preschool children, $t(22) = 2.59$, $p = .05$. For Grade 1, the continuous condition was characterized by a higher linear fit compared with both discrete and symbolic condition, $t(50) = 3.69$, $p = .002$, $t(50) = 4.36$, $p < .001$, respectively. Grade 3 children displayed a higher linear fit in the continuous and symbolic condition compared with the discrete condition, $t(88) = 2.8$, $p = .019$, $t(88) = 3.9$, $p < .001$, respectively.

Table 2
Type of Mapping Deployed in the Symbolic Condition as a Function of the Type of Mapping Observed in the Discrete Condition

		Type of mapping	
Grade	Discrete	Logarithmic	Linear
Preschool	Logarithmic	53.8	0.0
	Linear	34.6	11.5
Grade 1	Logarithmic	49.1	5.5
	Linear	32.7	12.7
Grade 3	Logarithmic	15.2	25.0
	Linear	10.9	48.9

Note. Cell values are percentages of children for each grade.

Discussion

In Experiment 1, we directly compared performances of children, from preschool to Grade 3, on three estimation tasks in which the quantities to be placed were continuous, discrete and symbolic. Crucially, in the discrete condition children could rely either on the spatial extent or on numerosity to estimate the position of the item. If children relied on the former, the pattern of estimates in the discrete condition should have mirrored those of the continuous condition. Conversely, if they relied on numerosity, the pattern of estimates should have resembled that of the symbolic condition.

Our results highlighted the presence of two distinct strategies: Approximately half of the children spontaneously encoded discrete quantities as numerosities despite the availability of continuous visual cues. The other half of the children seemed to rely on spatial extent to perform the task, thereby showing a more accurate mapping that mirrored the continuous quantity condition. For the continuous condition, children displayed a linear mapping already in preschool. Indeed, median estimates were better fit by the linear model than the logarithmic model for all groups, and at individual level only 6% of children were categorized as logarithmic in the continuous condition. The accuracy in positioning the items showed a slight improvement between preschool and Grade 3 children. These results can be explained by the fact that the continuous condition requires a simple transformation within the visuospatial domain, thereby yielding an unbiased performance even in the youngest group. As already observed in proportional judgments (Spence, 1990), the pattern of estimates for the continuous items presented a slight underestimation of small quantities. Although the vast majority of children displayed a linear mapping in the continuous condition, half of them showed a biased (log-like) pattern of estimation in the discrete condition. It is interesting that small discrete quantities were overestimated despite the availability of a strong continuous visual cue (cumulative surface) which could promote a direct visuospatial strategy. The overestimation of small numerosities strongly suggests that the stimuli were encoded as numerical magnitudes and then mapped onto the visual line similarly to their symbolic counterparts. This interpretation is in line with a study by Cordes and Brannon (2009), showing that numerosity can be more salient than other available continuous visual cues (also see Cantlon et al., 2010). Results also show that the proportion of children displaying a linear mapping in the discrete condition increased between Grades 1 and 3. Although some studies have found that educated adults still deploy a compressed mapping when estimating discrete quantities (Dehaene et al., 2008; Núñez et al., 2011), a recent study by Anobile et al. (2012) reported a linear mapping. In our study, the linear mapping can also be explained by the fact that spatial extent was perfectly correlated with numerosity, thereby allowing a much finer estimation of numerosity or even strategic reliance on continuous mapping in older children. Although the visuospatial properties of the discrete condition facilitated estimation, the analysis of the linear fit showed that even third graders were less linear in the discrete condition compared with both continuous and symbolic conditions. Discrete quantity estimation was further investigated in Experiment 2. Accordingly, further discussion of the interindividual differences in this condition is postponed to the General Discussion section.

Experiment 2

A limitation of Experiment 1 is that the type of quantity in the continuous condition was spatial in nature. Therefore, the linear pattern of estimates observed for continuous condition in Experiment 1 could be related to the spatial nature of the stimuli and might not be fully representative of continuous quantity estimation.

The main aim of Experiment 2 was to supplement the tasks used in Experiment 1 (see Figure 5, Panels a–c) with a novel nonspatial continuous condition to confirm that children deploy a linear

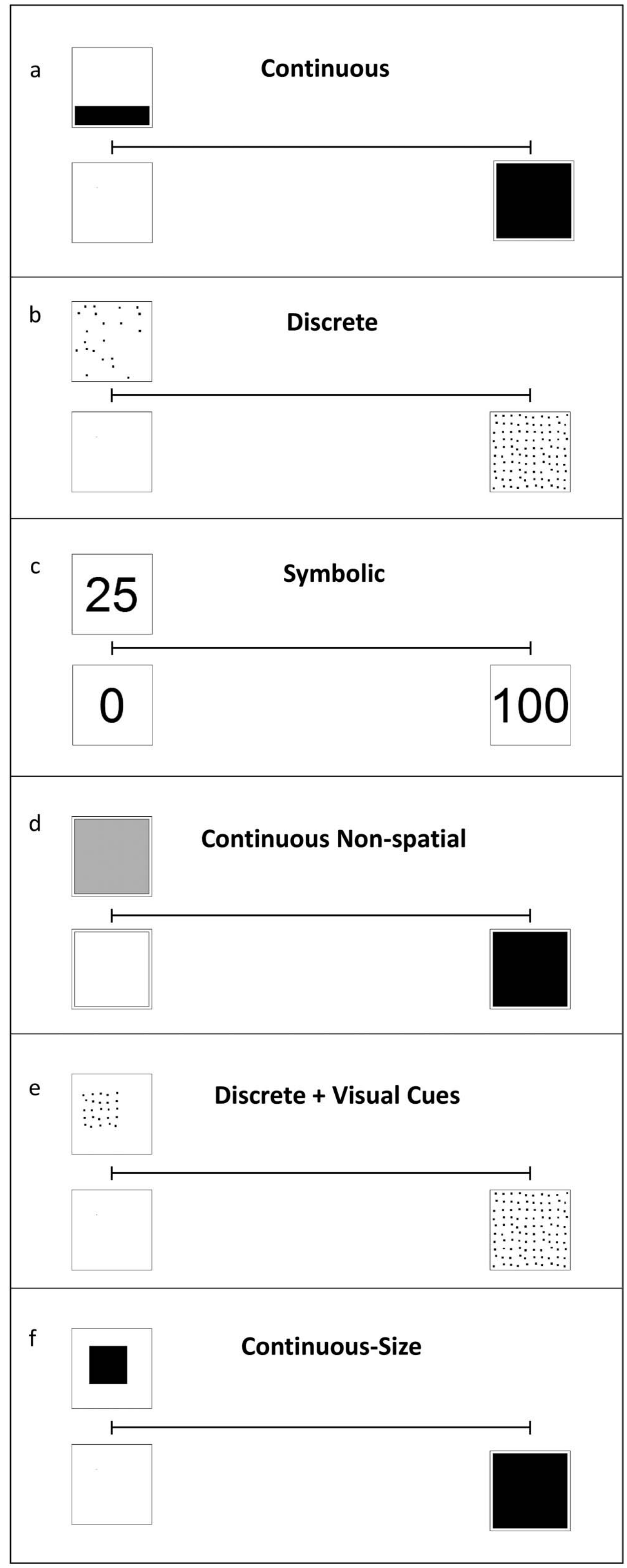

Figure 5. Example of six trials with the same quantity (i.e., 25) presented in the six different conditions: (a) continuous, (b) discrete, (c) symbolic, (d) continuous nonspatial, (e) discrete + visual cues, and (f) continuous size.

pattern in the estimation of continuous quantities, even in the absence of a spatial feature. The new continuous nonspatial condition (see Figure 5, Panel d) required to position different shades of gray on a line delimited by a white and a black square. Moreover, we included additional variants of the discrete and continuous (spatial) conditions to further investigate how the estimation strategy is influenced by continuous visual cues. In particular, we designed a new discrete condition so that numerosity, cumulative area and perimeter (occupied area) of the target items represented the same proportion of the full box (discrete + visual cues; see Figure 5, Panel e). This allowed us to assess whether the perfect correlation between numerosity and continuous visual cues (cumulative area and perimeter) would increase the reliance on a nonnumerical visual strategy, thereby yielding a linear pattern of estimates. We also designed a new continuous condition (continuous size; see Figure 5, Panel f) that was perfectly matched to the new discrete + visual cues condition. To this end, target quantities were black squares or rectangles that were identical in size (both width and height) to the items of the discrete + visual cues condition. Given that preschool and Grade 1 children displayed a similar pattern of estimates across conditions in Experiment 1, Experiment 2 was administered only to new samples of Grade 1 and Grade 3 children.

We predicted that children from both age groups would display a linear mapping in the continuous nonspatial condition, even if their accuracy in placing target quantities might decrease as a consequence of the lack of spatial landmarks (Vogel, Grabner, Schneider, Siegler, & Ansari, 2013). Regarding the discrete, continuous, and symbolic condition, we expected to replicate the results of Experiment 1. Grade 1 children should show less linearity in the discrete and symbolic conditions, whereas their mapping should be linear in the continuous condition. Grade 3 children, instead, should globally display a linear mapping across conditions, as in Experiment 1. Nevertheless, we expected them to show less linearity in the discrete condition compared with the continuous and the symbolic conditions. Regarding the discrete + visual cues condition, we expected children from both groups to show a linear mapping because the presence of a perfect correlation between numerosity and continuous visual cues (cumulative area and perimeter) should favor the reliance on a nonnumerical visual strategy, thereby yielding a linear pattern of estimates. Finally, the continuous-size condition requires children to compute the area of squares and rectangles, which vary in width and height, to provide a correct mapping. Consequently, this condition might be more difficult compared with the original continuous condition, in which target quantities only varied in height. Therefore, we predicted a linear mapping for both age groups, but with a reduced accuracy in positioning the quantities.

Method

Participants. Sixty-seven children from Grade 1 and Grade 3 were recruited from a school located in northern Italy. All children spoke Italian as first language and they were mostly of middle socioeconomic status. There were 27 children from Grade 1 (15 boys, $M_{\text{months}} = 79$, $SD = 3$) and 40 children from Grade 3 (18 boys, $M_{\text{months}} = 104$, $SD = 6$).

Procedure. Children completed a computerized version of the task in the school's computer room. The six different estimation conditions were randomly administered except for the symbolic condition which was always presented last. This order of presentation was intended to prevent children from understanding that the nonsymbolic items were equivalent to the symbolic numbers. Estimation tasks were presented as games, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Children were free to stop at any time. The task instructions were given by the experimenter before starting the task and children could ask questions if they had doubts.

Tasks. For all six conditions, a 20.3-cm long black line was presented in the center of the monitor (15-in. monitors with 1024 × 768 pixel resolution). In all continuous conditions, a white box was placed just below the left end of the line, whereas a full black box was placed just below the right end (see Figure 5). Target items were all presented in the upper left corner of the screen. For the continuous nonspatial condition, the item to be positioned was represented by a gray-shaded box in the upper left corner. Shades were calculated as percentages of black color using the red-green-blue (RGB) parameters (black: R = 0, G = 0, B = 0; white: R = 255, G = 255, B = 255). For the continuous condition, the quantity was represented by a partially filled box (as in Experiment 1). In the continuous-size condition, the quantity was represented by a black square or rectangle whose area corresponded to a percentage of the total area occupied by the reference full black box. In the two discrete conditions, the right-end reference quantity was a box filled with 100 small black squares (as in Experiment 1). For the discrete + visual cues condition, the quantity to be positioned was represented by a box filled with a variable number of small squares. Numerosity, cumulative area, and perimeter were perfectly correlated and represented the same proportion of the full box reference quantity. In the discrete condition, the quantity to be positioned was represented by a box filled with a variable number of randomly spread small squares. In the continuous-size, discrete + visual cues, and discrete conditions, each target quantity was randomly selected among a pool of 10 images. In the two discrete conditions, the reference quantity of 100 little squares was randomly selected among a pool of 10 images in which the spatial arrangement of the squares was jittered. In the symbolic condition, the left end of the line was labeled 0 and the right end was labeled 100. The number to be placed was presented in a box in the left upper corner. In all six conditions, children were required to estimate the position of the same quantities (i.e., 1, 2, 4, 6, 9, 12, 16, 25, 36, 42, 49, 56, 64, 72, 81). The target quantities were randomly presented within each estimation condition. Children placed the quantities on the line by moving an arrow using the mouse and clicking a mouse button to confirm the selected position. The movement of the arrow was bounded to the horizontal line. After pressing the mouse button, a red dot appeared on the selected location as feedback. Then, children pressed the key "s" to start another trial. The experiment was presented as a game. At the beginning of each condition, the experimenter gave instructions and explained what the child would see. Two practice trials were then administered to the child to verify that she had understood the task. The two practice trials in all conditions were represented by an empty box (0 in the symbolic condition) and by a full box (100 in the symbolic condition). The experimenter repeated the instructions if necessary, then each child completed the entire task without receiving any feedback.

Group analysis. Estimation accuracy was assessed using the PAE for each participant and condition (see Figure 6). A mixed ANOVA was computed with grade as between-subjects factor

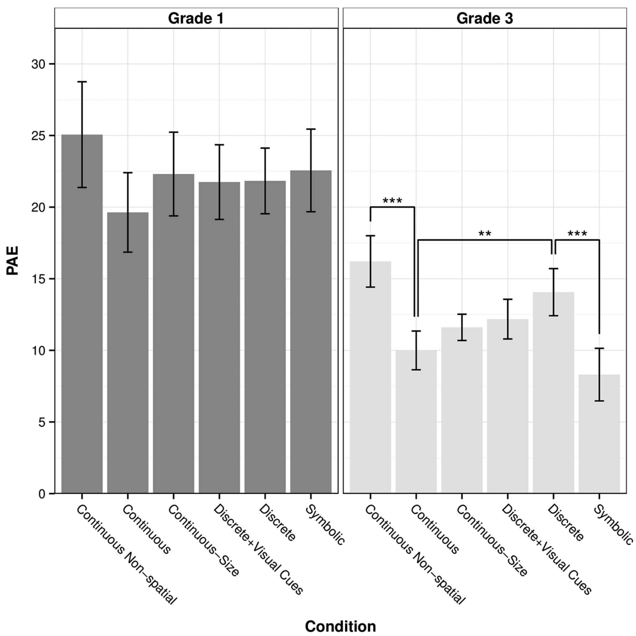

Figure 6. Percentage of absolute error (PAE) as a function of grade and condition. Bars represent within-subjects 95% confidence interval (Morey, 2008). All *p*s are Bonferroni-corrected. ** $p < .01$. *** $p < .001$.

(Grade 1 and Grade 3) and condition as within-subject factor (continuous nonspatial, continuous, continuous size, discrete + visual cues, discrete, symbolic). Mean PAEs, for Grade 1 and Grade 3 were 25%, and 16% in the continuous nonspatial condition, 20%, and 10% in the continuous condition, 22%, and 12% in the continuous-size condition, 22%, and 12% in the discrete + visual cues condition, 22%, and 14% in the discrete condition, 23%, and 8% in the symbolic condition. The main effect of condition $F(5, 325) = 7.84$, $MSE = 34.95$, $p < .001$, $\eta_g^2 = 0.07$, and the main effect of grade, $F(1, 65) = 82.38$, $MSE = 120.45$, $p < .001$, $\eta_g^2 = 0.34$, were both significant. Because the interaction was also significant, $F(5, 325) = 2.32$, $MSE = 34.95$, $p = .043$, $\eta_g^2 = 0.02$, we performed separate repeated-measures ANOVAs for each grade with condition as within-subject factor. Condition was significant only for Grade 3 children showing a difference in precision of estimation as a function of condition, $F(5, 195) = 14.08$, $MSE = 22.61$, $p_{[GG]} < .001$, $\eta_g^2 = 0.21$. Planned comparisons included the same contrasts performed in Experiment 1 (i.e., continuous vs. discrete, discrete vs. symbolic, discrete vs. symbolic) and two additional contrasts involving the new conditions, that is continuous versus continuous nonspatial and discrete + visual cues versus continuous size. The *t* test comparisons (we reported Bonferroni-adjusted *p*s for 5 comparisons) revealed that for continuous nonspatial condition the PAE was higher compared with continuous, $t(39) = 5.56$, $p < .001$. For the continuous condition the PAE was higher compared with the discrete condition, $t(39) = 3.71$, $p = .003$ but similar to the symbolic condition, $t(39) = 1.53$, $p = .673$. The continuous-size condition and the discrete + visual cues displayed a similar PAE, $t(39) = 0.79$, $p = 1$, whereas the PAE in the symbolic condition was lower compared with the discrete condition, $t(39) = 5.02$, $p < .001$.

As for Experiment 1, to understand the pattern of estimates for each condition, we first fit group medians and then individual data. The best fitting model was assessed following the same procedure as for Experiment 1. The linear model was significantly better than the logarithmic model in both groups for all conditions except for the symbolic condition. For the symbolic condition the logarithmic model was significantly better for Grade 1, whereas the linear model was significantly better for Grade 3 (see Figure 7). In the continuous nonspatial condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for both groups, Grade 1: $t(14) = 2.61$, $p = .02$, linear $R^2 = 94\%$ versus log $R^2 = 84\%$; Grade 3: $t(14) = 6.15$, $p < .001$, linear $R^2 = 99\%$ versus log $R^2 = 81\%$. In the continuous condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for both groups, Grade 1: $t(14) = 4.79$, $p < .001$, linear $R^2 = 97\%$ vs. log $R^2 = 80\%$; Grade 3: $t(14) = 5.31$, $p < .001$, linear $R^2 = 99\%$ versus log $R^2 = 76\%$. In the continuous-size condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for both groups, Grade 1: $t(14) = 2.64$, $p = .02$, linear $R^2 = 93\%$ versus log $R^2 = 81\%$; Grade 3: $t(14) = 4.51$, $p < .001$, linear $R^2 = 98\%$ versus log $R^2 = 74\%$. In the discrete + visual cues condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for both groups, Grade 1: $t(14) = 2.52$, $p = .024$, linear $R^2 = 97\%$ vs. log $R^2 = 82\%$; Grade 3: $t(14) = 5.49$, $p < .001$, linear $R^2 = 99\%$ versus log $R^2 = 79\%$. In the discrete condition, the linear model had the highest R^2 and was significantly different from the logarithmic model for both groups, Grade 1: $t(14) = 3.4$, $p = .004$, linear $R^2 = 96\%$ versus log $R^2 = 82\%$; Grade 3: $t(14) = 3.71$, $p = .002$, linear $R^2 = 98\%$ versus log $R^2 = 85\%$. In the symbolic condition, the logarithmic model had the highest R^2 and was significantly different from the linear model for Grade 1 whereas the linear model had the highest R^2 and was significantly different from the logarithmic model for Grade 3, Grade 1: $t(14) = 2.57$, $p = .022$, linear $R^2 = 71\%$ versus log $R^2 = 91\%$; Grade 3: $t(14) = 5.07$, $p < .001$, linear $R^2 = 98\%$ versus log $R^2 = 79\%$.

Individual analysis. We fit the linear and the logarithmic model on individual estimates for each condition as for Experiment 1. Table 3 shows the percentages of children with each type of mapping for each condition. For first graders, the percentage of children classified as linear tends to be higher compared with the percentage of those classified as logarithmic, except for the symbolic condition in which the log-like pattern is predominant. For third graders, linear mapping appears to be predominant across all conditions. Nevertheless, 35% of the third graders showed a logarithmic mapping in the discrete condition. The distribution of individual patterns of estimation was further analyzed by computing the R^2 of the linear fit.

R^2 of linear fit. We analyzed the linear R^2 for each child who displayed logarithmic or linear mapping in all the conditions (i.e., excluding those classified as "none"). There were 10 children from Grade 1 and 35 from Grade 3. We analyzed the linear R^2 in a mixed ANOVA with grade as between-subjects factor (Grade 1 and Grade 3) and condition as within-subject factor (continuous nonspatial, continuous, continuous-size, discrete + visual cues, discrete, symbolic, see Figure 8). The main effect of grade and the interaction Grade × Condition reached significance, $F(1, 43) = 32.85$, $MSE = 0.05$, $p < .001$, $\eta_g^2 = 0.19$; $F(5, 215) = 3.8$, $MSE =$

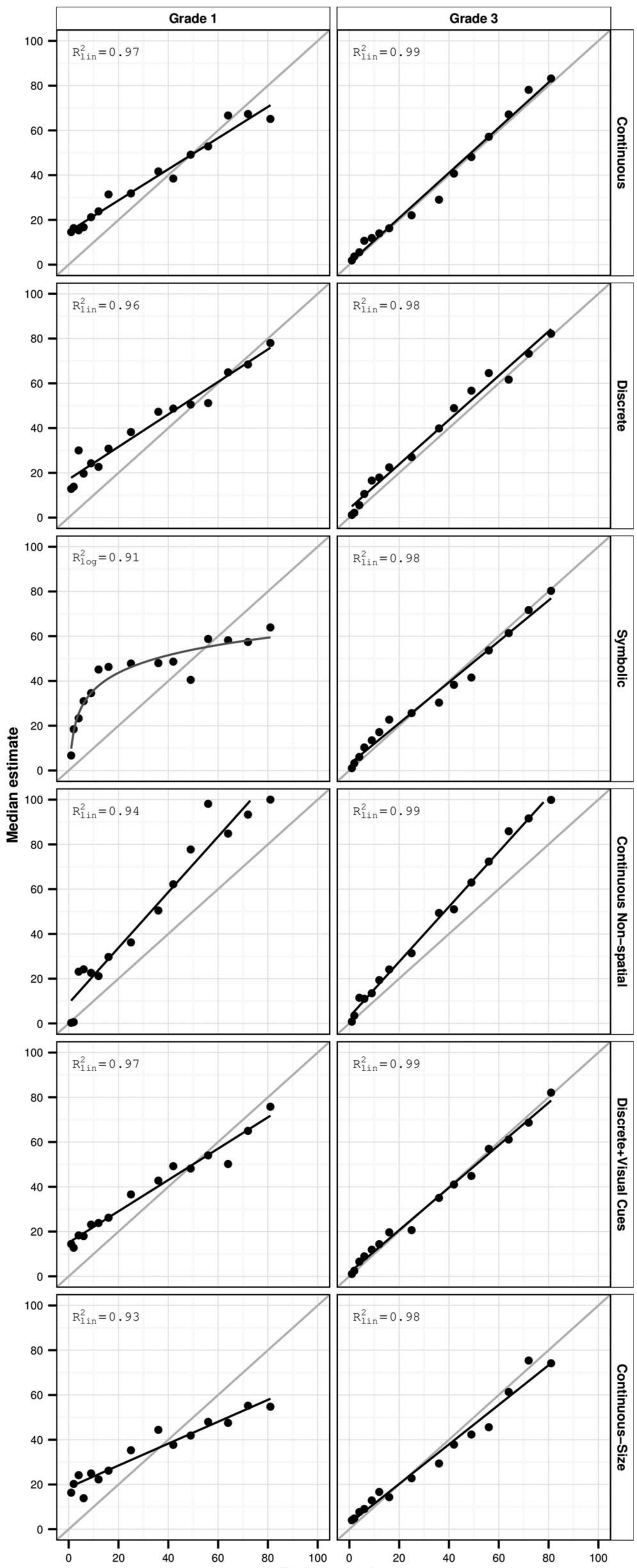

Figure 7. Median estimates and best fitting models as a function of grade for continuous nonspatial, continuous, continuous-size, discrete + visual cues, discrete and symbolic conditions.

Table 3
Type of Mapping in the Two Age Groups for Each Condition

Grade and condition	Type of mapping: None	Logarithmic	Linear
Grade 1			
Continuous nonspatial	18.5	25.9	5.6
Continuous	25.9	18.5	5.6
Continuous size	29.6	11.1	9.3
Discrete + visual cues	29.6	29.6	0.7
Discrete	14.8	29.6	5.6
Symbolic	37.0	44.4	8.5
Grade 3			
Continuous nonspatial	0.0	22.5	77.5
Continuous	2.5	7.5	90.0
Continuous size	0.0	2.5	97.5
Discrete + visual cues	7.5	17.5	75.0
Discrete	0.0	35.0	65.0
Symbolic	2.5	15.0	82.5

Note. Cell values represent percentages of children with row sums equal to 100%.

0.02, $p = .003$, $\eta^2_g = 0.06$, respectively. Separate repeated-measures ANOVAs for each grade with condition as within-subject factor were then run. Condition was significant only for Grade 3, $F(5, 170) = 4.68$, $MSE = 0.02$, $p < .001$, $\eta^2_g = 0.08$. We then carried out the same set of planned comparisons performed in the PAE analysis (i.e., continuous vs. discrete, discrete vs. symbolic, discrete vs. symbolic, continuous vs. continuous nonspatial, discrete + visual cues vs. continuous size). The *t* test comparisons (we reported Bonferroni-adjusted *p*s for five comparisons) revealed that the linear fit was highest in the symbolic condition and significantly different compared with the discrete condition,

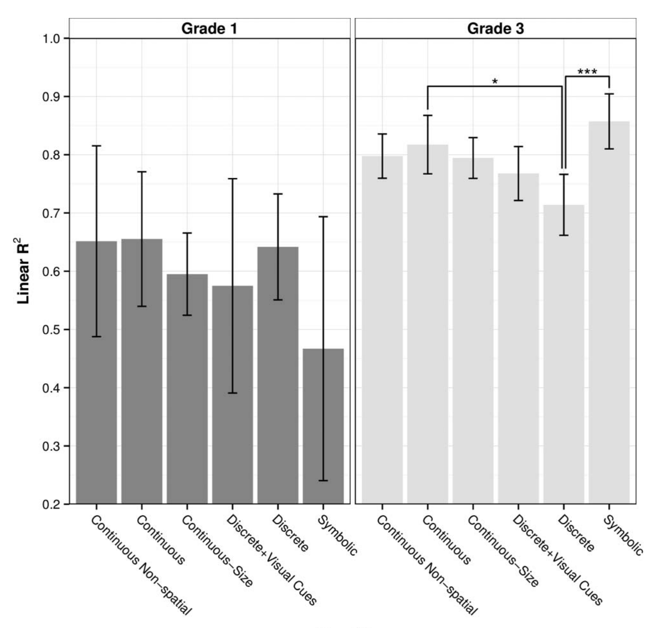

Figure 8. Linear R^2 as a function of grade for continuous nonspatial, continuous, continuous-size, discrete + visual cues, discrete and symbolic conditions. All *p*s are Bonferroni-corrected. * $p < .05$. *** $p < .001$.

$t(34) = 4.09$, $p = .001$. The linear fit was also higher in the continuous condition compared with the discrete condition, $t(34) = 3.08$, $p = .02$. The other comparisons did not reach statistical significance.

Discussion

In Experiment 2, children completed the same continuous, discrete, and symbolic conditions used in Experiment 1. Moreover, children also completed a continuous nonspatial condition to rule out that the linear pattern of estimates observed in Experiment 1 was due to the spatial nature of the continuous quantity task. We also added the discrete + visual cues condition to investigate whether perfect correlation between numerosity, cumulative area and perimeter would increase the reliance on a nonnumerical visual strategy, thereby yielding a performance similar to the matched continuous-size condition. The data from Grade 3 children perfectly replicated those of Experiment 1. In the continuous condition, children displayed a linear mapping and a high accuracy in positioning (i.e., low PAE). In the continuous nonspatial condition, 22.5% of Grade 3 children displayed a logarithmic mapping, however, at individual level, the R^2 of the linear fit was comparable to that of the continuous condition. We conclude that the mapping of the continuous nonspatial condition is basically linear, despite a tendency to overestimate all target quantities as indexed by a lower accuracy compared with the continuous condition. Therefore, this result demonstrates that the linearity of mapping for continuous quantity estimation is not tied to the spatial dimension of the target stimuli. A reduced accuracy in the mapping of continuous nonspatial quantities has been observed in a previous study on adult participants using a brightness estimation task (Vogel et al., 2013). Also in the continuous-size condition the pattern of estimates was characterized by a linear mapping. In summary, all conditions that entailed the mapping of continuous quantities onto a line were performed in a linear manner by the large majority of children. In the discrete condition, the linearity of estimation was significantly reduced in comparison to both symbolic and continuous conditions, thereby mirroring the results of Experiment 1. It is interesting that the discrete + visual cues condition and the matched continuous-size condition did not differ in terms of linearity, thereby suggesting that the availability of perfectly reliable visual cues (i.e., cumulative area and perimeter) promoted a mapping strategy based on continuous quantity rather than on discrete numerosity.

In contrast to third graders, our sample of Grade 1 children demonstrated some difficulties in completing one or more conditions of the estimation tasks. Indeed, only 10 children displayed a meaningful mapping (i.e., logarithmic or linear) across all conditions. Therefore, the conclusions that can be drawn for Grade 1 children are limited. One possible explanation for the discrepancy with Experiment 1, in which Grade 1 children successfully performed across all conditions, is presentation format. In Experiment 1, the task was administered paper-and-pencil as in previous studies (Siegler & Opfer, 2003) whereas in Experiment 2 tasks were presented on computers. Therefore it is conceivable that younger children experienced difficulties with the unfamiliar setting such as coordinating the mouse movements to respond (i.e., mouse pointing and clicking).

General Discussion

Results from the two experiments show that continuous spatially defined estimation tends to yield linear mapping. These tasks entail simple visual transformations most likely calling upon a proportional judgment process (Spence, 1990). Children estimated the size of the target item and then compared its relative size to the full box by selecting a position on the line. The computation of items surface and their relative proportional judgment seems to be an accurate process already in preschool children. In line with this result, children from 3 to 6 years of age displayed a better performance in comparing total area of stimuli (continuous dimension) than comparing their numerosity (discrete dimension; Odic, Libertus, Feigenson, & Halberda, 2013). The estimation of nonspatial continuous quantities (i.e., shades of gray) showed a linear pattern with higher percentage of error than for continuous spatial quantity estimation. A possible explanation for the latter difference is that in the spatial condition the outer box provides a reference for judging the target quantity and it is relatively easy to identify the midpoint (i.e., a half-full box). Conversely, the nonspatial continuous condition lacks any explicit cue about the gray level corresponding to the middle of the grayscale. A relatively low estimation accuracy in grayscale estimation was observed in the study of Vogel et al. (2013) on adult participants.

Regardless of the estimation accuracy, the crucial finding of the present study is that the pattern of estimates in all the nonnumerical quantity estimation tasks was markedly linear, which is in sharp contrast with the nonlinear pattern observed in the numerical estimation tasks. For example, both preschoolers and first graders in Experiment 1 showed a linear pattern of estimates for continuous quantities, whereas they displayed a nonlinear positioning in the numerical conditions, whether symbolic or nonsymbolic/discrete. It is worth reiterating that the target quantities were the same across all conditions, which implies that the nonlinear pattern observed in the numerical condition cannot be attributed to confounds in the stimuli (e.g., oversampling of small numbers in the stimulus set, Barth & Paladino, 2011) or to the choice of a specific fitting function. As noted in the introduction, the present results are agnostic to the issue of which function best describes the pattern of estimates in the symbolic condition (Barth & Paladino, 2011; Cohen & Sarnecka, 2014; Ebersbach et al., 2008; Moeller et al., 2009; Opfer et al., 2011). Whatever the outcome of the ongoing debate, our interpretation of the results is based on the different patterns of estimates observed *within subjects* between numerical and nonnumerical quantity estimations. Moreover, the comparison across age groups clearly confirmed the well-known developmental trajectory of numerical estimation (Berteletti et al., 2010; Booth & Siegler, 2006; Siegler & Opfer, 2003), whereas nonnumerical (i.e., continuous) quantity estimation followed a different trend and it was not characterized by a drastic change in the pattern of estimates during the time window investigated in the present study.

With regard to the numerical estimation conditions, we observed both similarities and differences between symbolic and discrete quantity estimation tasks. The analysis of individual patterns of estimates suggested that in the discrete condition some children spontaneously encoded the discrete stimuli as numerical quantities despite the availability of visual cues (Cantlon et al., 2010; Hannula-Sormunen, 2014), in line with the hypothesis that numerosity is a primary visual property (Burr & Ross, 2008; Stoianov &

Zorzi, 2012). These children displayed a biased and less linear mapping (also see Dehaene et al., 2008; Núñez et al., 2011), thereby confirming that the encoding of numerical information does not rely on the same process implemented for continuous quantity estimation (also see Odic et al., 2013). In line with this result, the processing of continuous (i.e., area, brightness) and discrete (i.e., numerosity) quantities have been found to be processed in adjacent but distinct brain areas (Castelli, Glaser, & Butterworth, 2006; Vogel et al., 2013; for a review, Cohen Kadosh, Lammertyn, & Izard, 2008). Moreover, computational modeling has shown that numerosity is a higher order summary statistics compared with a continuous quantity like cumulative area (Stoianov & Zorzi, 2012). Nevertheless, the estimation pattern appeared to be more linear when the visual cues (i.e., perimenter and cumulative area) were made more salient by perfectly correlating with numerosity (i.e., discrete + visual cues condition in Experiment 2).

Future research might investigate the source of differences in individual strategy when children estimate discrete quantities characterized by varying physical dimensions (e.g., cumulative area, perimeter). The ability to accurately extract numerical information in complex conditions (e.g., when the correlation between physical dimensions and numerosity is unreliable) may be crucial in explaining the relation between numerosity comparison and mathematical skills (Cappelletti, Didino, Stoianov, & Zorzi, 2014; Fuhs & McNeil, 2013; Gilmore et al., 2013). It is also worth noting that the tendency of some children to spontaneously focus on numerosity (Hannula & Lehtinen, 2005; Hannula, Rasanen, & Lehtinen, 2007; Sella, Berteletti, Lucangeli, & Zorzi, 2015) is a plausible source of the interindividual variability observed in the discrete condition. The ability to focus on numerosity has been reported to be a distinct attentional component as well as a predictor of mathematical achievement in the early stages of development (Hannula, Lepola, & Lehtinen, 2010). More broadly, children who spontaneously focus on numerical aspects of their environment have more opportunities to refine their number skills (e.g., school, home) and they may be more prone to immediately perceive the discrete stimuli employed in our study as numerosities (Hannula-Sormunen, 2014; Sella, Berteletti, Lucangeli, & Zorzi, 2015).

In conclusion, our results show that estimations of numerical and nonnumerical quantities rely on different processing strategies and follow different developmental trajectories. In a broader context, our findings speak to the issue of whether numerical quantity is processed by a common quantity system (Walsh, 2003) or has special status in the human neurocognitive system (e.g., Zorzi, Di Bono, & Fias, 2011; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006; see Cohen Kadosh et al., 2008, for review). Together with a previous study highlighting the developmental trajectories in the spatial mapping of numerical and nonnumerical order (Berteletti et al., 2012), the present study suggests that numerical estimation is indeed a special case of quantity estimation.

References

Agrillo, C., Dadda, M., Serena, G., & Bisazza, A. (2009). Use of number by fish. *PLoS ONE, 4,* e4786. http://dx.doi.org/10.1371/journal.pone.0004786

Anobile, G., Cicchini, G. M., & Burr, D. C. (2012). Linear mapping of numbers onto space requires attention. *Cognition, 122,* 454–459. http://dx.doi.org/10.1016/j.cognition.2011.11.006

Bakeman, R. (2005). Recommended effect size statistics for repeated measures designs. *Behavior Research Methods, 37,* 379–384. http://dx.doi.org/10.3758/BF03192707

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14,* 125–135. http://dx.doi.org/10.1111/j.1467-7687.2010.00962.x

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology, 46,* 545–551. http://dx.doi.org/10.1037/a0017887

Berteletti, I., Lucangeli, D., & Zorzi, M. (2012). Representation of numerical and nonnumerical order in children. *Cognition, 124,* 304–313. http://dx.doi.org/10.1016/j.cognition.2012.05.015

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42,* 189–201. http://dx.doi.org/10.1037/0012-1649.41.6.189

Bouwmeester, S., & Verkoeijen, P. P. J. L. (2012). Multiple representations in number line estimation: A developmental shift or classes of representations? *Cognition and Instruction, 30,* 246–260. http://dx.doi.org/10.1080/07370008.2012.689384

Burr, D., & Ross, J. (2008). A visual sense of number. *Current Biology, 18,* 425–428. http://dx.doi.org/10.1016/j.cub.2008.02.052

Cantlon, J. F., & Brannon, E. M. (2006). Shared system for ordering small and large numbers in monkeys and humans. *Psychological Science: A Journal of the American Psychological Society, 17,* 401–406. http://dx.doi.org/10.1111/j.1467-9280.2006.01719.x

Cantlon, J. F., Safford, K. E., & Brannon, E. M. (2010). Spontaneous analog number representations in 3-year-old children. *Developmental Science, 13,* 289–297. http://dx.doi.org/10.1111/j.1467-7687.2009.00887.x

Cappelletti, M., Didino, D., Stoianov, I., & Zorzi, M. (2014). Number skills are maintained in healthy ageing. *Cognitive Psychology, 69,* 25–45. http://dx.doi.org/10.1016/j.cogpsych.2013.11.004

Castelli, F., Glaser, D. E., & Butterworth, B. (2006). Discrete and analogue quantity processing in the parietal lobe: A functional MRI study. *Proceedings of the National Academy of Sciences of the United States of America, 103,* 4693–4698. http://dx.doi.org/10.1073/pnas.0600444103

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10,* 408–411. http://dx.doi.org/10.1111/1467-9280.00177

Clearfield, M. W., & Mix, K. S. (2001). Amount versus number: Infants ' use of area and contour length to discriminate small sets. *Journal of Cognition and Development, 2,* 243–260. http://dx.doi.org/10.1207/S15327647JCD0203_1

Cohen, D. J., & Sarnecka, B. W. (2014). Children's number-line estimation shows development of measurement skills (not number representations). *Developmental Psychology, 50,* 1640–1652. http://dx.doi.org/10.1037/a0035901

Cohen Kadosh, R., Lammertyn, J., & Izard, V. (2008). Are numbers special? An overview of chronometric, neuroimaging, developmental, and comparative studies of magnitude representation. *Progress in Neurobiology, 84,* 132–147. http://dx.doi.org/10.1016/j.pneurobio.2007.11.001

Cordes, S., & Brannon, E. M. (2008). The difficulties of representing continuous extent in infancy: Using number is just easier. *Child Development, 79,* 476–489. http://dx.doi.org/10.1111/j.1467-8624.2007.01137.x

Cordes, S., & Brannon, E. M. (2009). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12,* 453–463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008, May 30). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science, 320,* 1217–1220. http://dx.doi.org/10.1126/science.1156540

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology, 99,* 1–17. http://dx.doi.org/10.1016/j.jecp.2007.08.006

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8,* 307–314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Field, A., Miles, J., & Field, Z. (2012). *Discovering statistics using R*. Thousand Oaks, CA: Sage.

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science, 16,* 136–148. http://dx.doi.org/10.1111/desc.12013

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33,* 277–299. http://dx.doi.org/10.1080/87565640801982361

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., . . . Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS ONE, 8,* e67374. http://dx.doi.org/10.1371/journal.pone.0067374

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction, 15,* 237–256. http://dx.doi.org/10.1016/j.learninstruc.2005.04.005

Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology, 107,* 394–406. http://dx.doi.org/10.1016/j.jecp.2010.06.004

Hannula, M. M., Rasanen, P., & Lehtinen, E. (2007). Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration. *Mathematical Thinking and Learning, 9,* 51–57. http://dx.doi.org/10.1080/10986060709336605

Hannula-Sormunen, M. (2014). Spontaneous focusing on numerosity and its relation to counting and arithmetic. In R. Cohen-Kadosh & A. Dowker (Eds.), *Handbook of numerical cognition* (pp. 37–41). New York, NY: Oxford University Press. http://dx.doi.org/10.1093/oxfordhb/9780199642342.013.018

Hollands, J. G., & Dyre, B. P. (2000). Bias in proportion judgments: The cyclical power model. *Psychological Review, 107,* 500–524. http://dx.doi.org/10.1037/0033-295X.107.3.500

Karolis, V., Iuculano, T., & Butterworth, B. (2011). Mapping numerical magnitudes along the right lines: Differentiating between scale and bias. *Journal of Experimental Psychology: General, 140,* 693–706. http://dx.doi.org/10.1037/a0024255

Lawrence, M. A. (2013). *ez: Easy analysis and visualization of factorial experiments* (R Package Version 4.2-2) [Computer software]. Retrieved from http://CRAN.R-Project.org/package=ez

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin, 128,* 278–294. http://dx.doi.org/10.1037/0033-2909.128.2.278

Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology, 103,* 503–515. http://dx.doi.org/10.1016/j.jecp.2009.02.006

Morey, R. D. (2008). Confidence intervals from normalized data: A correction to Cousineau (2005). *Tutorials in Quantitative Methods for Psychology, 4,* 61–64.

Núñez, R., Doan, D., & Nikoulina, A. (2011). Squeezing, striking, and vocalizing: Is number representation fundamentally spatial? *Cognition, 120,* 225–235. http://dx.doi.org/10.1016/j.cognition.2011.05.001

Odic, D., Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Developmental change in the acuity of approximate number and area representations. *Developmental Psychology, 49,* 1103–1112. http://dx.doi.org/10.1037/a0029472

Opfer, J. E., Siegler, R. S., & Young, C. J. (2011). The powers of noise-fitting: Reply to Barth and Paladino. *Developmental Science, 14,* 1194–1204. http://dx.doi.org/10.1111/j.1467-7687.2011.01070.x

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences, 14,* 542–551. http://dx.doi.org/10.1016/j.tics.2010.09.008

R Development Core Team. (2013). *R: A language and environment for statistical computing*.

Sella, F., Berteletti, I., Lucangeli, D., & Zorzi, M. (2015). Spontaneous non-verbal counting in toddlers. *Developmental Science*. Advance online publication. http://dx.doi.org/10.1111/desc.12299

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75,* 428–444. http://dx.doi.org/10.1111/j.1467-8624.2004.00684.x

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14,* 237–250. http://dx.doi.org/10.1111/1467-9280.02438

Siegler, R. S., Thompson, C., & Opfer, J. E. (2009). The logarithmic-to-linear shift: One learning sequence, many tasks, many time scales. *Mind, Brain, and Education, 3,* 143–150. http://dx.doi.org/10.1111/j.1751-228X.2009.01064.x

Spence, I. (1990). Visual psychophysics of simple graphical elements. *Journal of Experimental Psychology: Human Perception and Performance, 16,* 683–692. http://dx.doi.org/10.1037/0096-1523.16.4.683

Stoianov, I., & Zorzi, M. (2012). Emergence of a "visual number sense" in hierarchical generative models. *Nature Neuroscience, 15,* 194–196. http://dx.doi.org/10.1038/nn.2996

Verguts, T., Fias, W., & Stevens, M. (2005). A model of exact small-number representation. *Psychonomic Bulletin & Review, 12,* 66–80. http://dx.doi.org/10.3758/BF03196349

Vogel, S. E., Grabner, R. H., Schneider, M., Siegler, R. S., & Ansari, D. (2013). Overlapping and distinct brain regions involved in estimating the spatial position of numerical and nonnumerical magnitudes: An fMRI study. *Neuropsychologia, 51,* 979–989. http://dx.doi.org/10.1016/j.neuropsychologia.2013.02.001

Walsh, V. (2003). A theory of magnitude: Common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences, 7,* 483–488. http://dx.doi.org/10.1016/j.tics.2003.09.002

Wickham, H. (2009). *ggplot2: Elegant graphics for data analysis*. New York, NY: Springer-Verlag.

Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In M. Hahn & S. Stoness (Eds.), *Twenty-First Annual Meeting of the Cognitive Science Society* (pp. 778–783). Mahwah, NJ: Erlbaum.

Zorzi, M., Di Bono, M. G., & Fias, W. (2011). Distinct representations of numerical and nonnumerical order in the human intraparietal sulcus revealed by multivariate pattern recognition. *NeuroImage, 56,* 674–680. http://dx.doi.org/10.1016/j.neuroimage.2010.06.035

Zorzi, M., Priftis, K., Meneghello, F., Marenzi, R., & Umiltà, C. (2006). The spatial representation of numerical and nonnumerical sequences: Evidence from neglect. *Neuropsychologia, 44,* 1061–1067. http://dx.doi.org/10.1016/j.neuropsychologia.2005.10.025

Zorzi, M., Stoianov, I., & Umiltà, C. (2005). Computational Modeling of Numerical Cognition. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 67–84). New York, NY: Psychology Press.

Received March 15, 2013
Revision received March 5, 2015
Accepted March 18, 2015 ■