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Spatial order relates to the exact numerical magnitude of digits in young children

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ABSTRACT

Spatial representation of numbers has been repeatedly associated with the development of numerical and mathematical skills. However, few studies have explored the contribution of spatial mapping to exact number representation in young children. Here we designed a novel task that allows a detailed analysis of direction, ordinality, and accuracy of spatial mapping. Preschool children, who were classified as competent counters (cardinal principle knowers), placed triplets of sequentially presented digits on the visual line. The ability to correctly order triplets tended to decrease with the larger digits. When triplets were correctly ordered, the direction of spatial mapping was predominantly oriented from left to right and the positioning of the target digits was characterized by a pattern of underestimation with no evidence of logarithmic compression. Crucially, only ordinality was associated with performance in a digit comparison task. Our results suggest that the spatial (ordinal) arrangement of digits is a powerful source of information that young children can use to construct the representation of exact numbers. Therefore, digits may acquire numerical meaning based on their spatial order on the number line.

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Introduction

Number words and numerals are sounds and symbols to which young children need to assign exact numerical meaning. In this vein, counting can be considered the first attempt to establish an association between number words and external numerical quantities. The ability to count correctly requires children to recite number words in the right order (stable order principle), associate only one element in the set with one number word (one-to-one correspondence), and understand that the last recited number word corresponds to the numerosity of the set (cardinality principle) (Gelman & Gallistel, 1978). According to the knower-level theory (Carey, 2001; Sarnecka & Carey, 2008; Wynn, 1990), children acquire the numerical meaning of number words during specific developmental stages. Initially, children lack the numerical meaning of the number words and, therefore, usually grab a handful of items when asked to collect a numerosity from a large set of objects (as in the Give-a-Number task; Wynn, 1990). Children sequentially acquire the numerical meaning of "one," "two," "three," and "four" as they correctly collect these numerical quantities when requested. Children at this stage are considered to be subset knowers because their knowledge of the numerical meaning of number words is still limited to a subset of the counting list. The limit of four elements resembles the number of items that can be simultaneously stored in memory by means of the object tracking system (OTS) (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; Sarnecka, 2015). The repeated association between the first number words and small numerosities represented via the OTS enables children to achieve a conceptual change (Carey, 2001, 2004) and an understanding that an additional counted object corresponds to the next number word in the counting list (i.e., successor function) (but see Cheung, Rubenson, & Barner, 2017). Children who grasp this concept can extend the cardinality principle to the entire counting list, thereby becoming cardinal principle knowers (CP-knowers). As a consequence of achieving the cardinality principle, all CP-knowers should be able to successfully choose the larger of two number words within their counting list. Surprisingly, this is not the case; the mastery of the cardinality principle does not imply a full understanding of the magnitude relation between numbers (Davidson, Eng, & Barner, 2012; Le Corre, 2014; Sella, Berteletti, Lucangeli, & Zorzi, 2017).

Le Corre (2014) showed that all CP-knowers can successfully compare pairs formed by two small (i.e., <4) number words or by a small number word and a large number word, conceivably because small number words can be easily mapped to the corresponding exact numerical quantities via the OTS. Only some CP-knowers are able to compare pairs formed by two large number words (i.e., >4; e.g., "six" vs. "ten"). These children, called CP-mappers as compared with CP-non-mappers, display a mapping between external numerical quantities and the counting list that it is not the byproduct of a counting routine. CP-mappers provide numerosity estimates that increase linearly with the number of objects (e.g., dots) in briefly presented visual sets (Le Corre, 2014; Le Corre & Carey, 2007). The ability to verbally estimate the numerosity of large sets of objects (i.e., >4) relies on the approximate number system (ANS), whereby numerical quantities are represented by Gaussian curves of activation that progressively overlap with increasing numerical magnitude (Feigenson et al., 2004; Izard & Dehaene, 2008; Piazza, 2010). In this regard, the ability to map from the ANS to number words conveys an understanding that later number words in the counting list are associated with larger numerical quantities (i.e., later-greater principle; Le Corre, 2014). Davidson et al. (2012) also showed that CP-knowers' performance in comparing number words ranging from 5 to 9 and from 21 and 27 was unexpectedly low. Again, CP-mappers performed better at comparing number words than CP-non-mappers. CP-mappers' use of the cardinality principle, as assessed by the Give-a-Number task, appears to be a routine where children have understood that the last recited number word is the answer to the question "How many?" (Sarnecka & Carey, 2008). It follows that the cardinality principle does not imply a deep understanding of the numerical magnitude associated with number words (i.e., semantic induction; Davidson et al., 2012).

Most of the research on the early development of exact number representations has focused on the acquisition of number word meaning. Less attention has been devoted to the understanding of Arabic numerals. In a recent study (Sella et al., 2017), CP-knowers were classified as mappers when they demonstrated the ability to linearly place digits on a visual "number line" from 1 to 10 (i.e., Number Line task [NL task]; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Opfer, 2003). Only

CP-mappers reliably chose the larger of two visually presented digits. Conversely, children demonstrating a non-meaningful placement of digits (i.e., CP-non-mappers) performed poorly when comparing digits. Therefore, the ability to map digits on a visual (or imaginary) line (i.e., spatial mapping principle; Sella et al., 2017) demonstrates an understanding of magnitude relations between digits.

In summary, the acquisition of the cardinality principle does not imply a full understanding of magnitude relations for symbolic numbers. Moreover, children seem to rely on different mapping strategies to assign numerical meaning to number words and Arabic numerals (Sella, Lucangeli, & Zorzi, 2018). The accurate mapping of large numerical quantities and number words in the counting list marks a better understanding of the numerical magnitude represented by number words. Similarly, the linear mapping of digits on the number line marks a better understanding of the exact numerical magnitude conveyed by numerals, at least in the interval from 1 to 9.

In the current study, we further investigated the understanding of symbolic numerical magnitude in CP-knowers in relation to the spatial mapping of numbers. In the study of Sella et al. (2017), the ability to map numbers onto space was evaluated using the NL task. In this task, children are presented with a horizontal line defining a numerical interval. The smaller number is presented on the left end of the line (e.g., 1), and the larger number is presented on the right end (e.g., 10). A target number is presented centrally above the line, and children are asked to mark the position of the target number on the line with a pencil or cursor of the mouse. Then children proceed to the following trial, which requires placing a new target on the same line but with no trace of previous marks. The proportion of absolute error (i.e., [|estimate-target number|/numerical interval]) is considered to be a reliable measure of children's number–space mapping accuracy (Siegler & Booth, 2004). In addition, Siegler and Opfer (2003) originally fit a linear and logarithmic model on the estimates as a function of target numbers, finding that children shift from a biased logarithmic to an accurate linear mapping of numbers with age and expertise (Berteletti et al., 2010; Siegler & Opfer, 2003; Siegler, Thompson, & Opfer, 2009). The shift from a biased logarithmic to linear mapping has been explained by different theoretical accounts, giving rise to a lively debate (Barth & Paladino, 2011; Cohen & Quinlan, 2018; Cohen & Sarnecka, 2014; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Opfer, Siegler, & Young, 2011; Slusser, Santiago, & Barth, 2013). Overall, the performance in the NL task has been found to correlate with other numerical tasks and more generally with math achievement (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Sella, Berteletti, Brazzolotto, Lucangeli, & Zorzi, 2014; for a meta-analysis, see Schneider et al., 2018), thereby making the task a useful tool for research and clinical purposes.

Despite its wide use, the NL task presents some limitations to a detailed analysis of the development of number–space association in young children. Placing numbers onto spatial locations on a line entails three distinct components: (a) ordinality (because the choice of spatial locations should reflect the ordering of numerical magnitudes, e.g., $n1 > n2 > n3$ and $n1 < n2 < n3$), (b) directionality (because moving in one spatial direction—left to right or right to left—implies an increase in numerical magnitude), and (c) spacing (because distances between locations reflect systematic changes in numerical magnitude, e.g., linear spacing). Formal assessment of these components would provide a detailed analysis of spatial mapping and assess their different association to symbolic magnitude representation in young children. The following paragraphs discuss three intrinsic limitations of the classic NL task that prevent detailed analysis that instigated the introduction of a novel experimental paradigm.

First, in the NL task, the smallest number is placed at the left end and the largest number is placed at the right end of the line, thereby forcing children to place numbers in a left-to-right direction. Preschool children already display a culturally constrained directionality (left to right in Western cultures) of the number–space association across a variety of symbolic numerical tasks such as counting direction (Briars & Siegler, 1984; Knudsen, Fischer, & Aschersleben, 2014; McCrink, Perez, & Baruch, 2017) and when comparing non-symbolic numerical quantities (Patro & Haman, 2012). Accordingly, young children display a better performance in the NL task when the direction is left to right compared with right to left (Ebersbach, 2015). However, the association between numbers and space is still flexible during the early stages of development, with some children lacking a definite directionality or even preferring to associate numbers in the non-canonical (right-to-left) direction (McCrink & Opfer, 2014; Patro, Nuerk, & Cress, 2016). The classic version of the NL task prevents the assessment of directionality, which may contribute to the understanding of magnitude relations between digits.

Second, the NL task implicitly assumes that children understand the numerical interval entailed by the bounded line. However, the numerical magnitude of the number located at the right end of the line may be obscure to young children, who might only understand the meaning of smaller numbers. This lack of knowledge may explain why some children place target numbers regardless of their numerical magnitude, for example, all the numbers placed in the middle of the line (Berteletti et al., 2010; Sella, Berteletti, Lucangeli, & Zorzi, 2015; Sella et al., 2017; Siegler & Ramani, 2009). It has also been proposed that children's limited numerical knowledge can account for the biased (loglike) mapping of target numbers on the line (i.e., familiarity model; Ebersbach et al., 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009; but see Thompson & Opfer, 2010). Smaller and more familiar numbers would be linearly placed on the left side of the line, whereas larger and less familiar numbers would be placed toward the middle point of the line, thereby resembling a compressed mapping. The same influence of familiarity could be applied to the interval from 1 to 10 that is usually adopted to explore early numerical competences (Berteletti et al., 2010). It is conceivable that some children do not know the meaning of the number 10 when presented with the number line from 1 to 10. These children may place all the numbers in the middle of the line or at the extreme without really understanding how to accomplish the task.

Third, in the NL task, each target number is mapped on a new line without taking into account whether the spatial order of target numbers is respected. Consider, for instance, a child who places the number 8 toward the end of the line from 1 to 10 and in the subsequent trial places the number 9 slightly to the left compared with the position in which 8 was placed. In this scenario, the child knows that 8 and 9 need to be placed on the rightmost part of the line, but it remains unclear whether the child knows that 9 should be placed farther to the right than 8. To address this issue Siegler and Ramani (2008) calculated an order index based on the comparison of the child's estimate for each target number with the estimate provided for each of the other target numbers. However, this order index does not take into account the effect that previous estimates can exert on the current estimate. To overcome this problem, Sullivan and Barner (2014) implemented a multiple estimate version of the NL task (0–100 interval) in which children used different color pencils to mark target numbers on the line. In this way, children could see their previous estimates and, therefore, adjust their current estimates accordingly. For 5-year-olds, the possibility to see their previous estimates improved the ability to correctly order numbers (ordinality was respected if the provided estimate was correctly spatially ordered in comparison with the previous one). This result implies that children who remember the positions of previous estimates may perform better in the classic version of the NL task (Sullivan & Barner, 2014). The lack of a precise assessment of ordinality and the possible effect of memory represent two other limitations of the NL task for the purpose of fine-grained analyses of the spatial mapping of numbers.

In this study, we aimed to investigate the relation between spatial mapping and the ability to compare Arabic digits in young children. We decided to investigate the performance of the digit comparison task because it directly assesses children's knowledge of symbolic numerical magnitude. Moreover, comparing the magnitude of digits is one of the most widely used tasks to assess basic symbolic numerical knowledge. In line with this view, the rapidity in comparing digits is a reliable predictor of future math achievement (see meta-analysis of Schneider et al., 2017). Preschoolers involved in the study had mastered the cardinality principle, understood the numerical meaning of "more," could verbally count up to 9 (at least), and could accurately move a mouse cursor. Children completed a novel computerized task capable of assessing direction, ordinality, and spatial accuracy—the Direction, Ordinality, and Space task (DOS task)—in the mapping of sets of three digits on a visual line. The presentation of a new triplet in every task prevented the recall of previous estimates from influencing the performance in the subsequent trials (Sullivan & Barner, 2014). The digit comparison task entailed the same triplets of digits used in the DOS task to allow a direct comparison of performance in the two tasks. Importantly, the sets of digits were always read by the experimenter to prevent any influence of number naming skills. Aiming to replicate and extend the findings of previous studies (Davidson et al., 2012; Le Corre, 2014), children also estimated the numerosity of briefly presented large visual sets to assess the correlation between numerosity estimation and symbolic comparison skills. Our hypothesis was that both the spatial mapping of numbers and numerosity estimation would predict digit comparison performance. Finally, the new spatial mapping task can provide a novel input to the debate regarding the nature of the compressed (loglike) spatial mapping of digits that has been repeatedly observed using the NL task because assessment of the type of mapping can be restricted to familiar and correctly ordered digits.

Method

Participants

A total of 69 children from two different nursery schools located in northeastern Italy took part in the study after informed consent was obtained by parents or legal guardians. The attendance at the school was not mandatory, but the children involved in the study attended most of the days of the week and were members of families with middle socioeconomic status (SES). Of this original sample, 6 children were excluded from the analyses for the following reasons: interrupting the testing session ($n = 3$), having a suspected cognitive disability as referred by the teacher ($n = 1$), and not complying with the task instructions ($n = 2$). In addition, 1 participant was removed after being classified as a subset-knower in the Give-a-Number task (see below). The final sample consisted of 62 children (31 boys; $M_{\text{age}} = 69$ months, $SD = 4$, range = 55–76). The study was approved by the psychological science ethics committee at the University of Padova.

Tasks

Simple dot comparison

Two numerical sets were presented on a landscape A4 sheet (approximately U.S. letter size), and children were asked to select the set with more dots. Children were not allowed to count the dots and were prompted to respond immediately. There were six comparisons (i.e., 12 vs. 24, 15 vs. 30, 8 vs. 16, 11 vs. 22, 9 vs. 18, and 12 vs. 24) presented sequentially that entailed the same numerical ratio (i.e., 1:2). All the presented sets contained dots over 4 to prevent the use of subitizing. Numerical sets were generated using the free software Panamath (Halberda, Ly, Wilmer, Naiman, & Germine, 2012). The two sets were presented in separated boxes on the left side (yellow dots) and right side (blue dots) of the sheet. In half of the trials the cumulative surface area of the dots in a set was proportional to the number of dots, whereas in the other half the cumulative surface area was matched across sets. This task was administered to ensure that children understood the meaning of "more numerous."

Forward enumeration

Children were asked to recite aloud the counting list starting from 1 and were stopped when they reached 50, when they could not go any further, or when they committed a mistake (e.g., skipping one or more numbers). Children were allowed to correct themselves immediately if they committed a mistake. For each child, the highest recited number was recorded.

Naming

Children were presented with an Arabic digit on a piece of cardboard (3.7×3.7 cm) and were asked to name it aloud. All the digits from 1 to 9 were presented in the following order: 3, 9, 2, 4, 7, 1, 5, 8, 6. One point was awarded for each correct naming, and the percentage of correct responses was calculated.

GaN

The task was adapted from Wynn's (1990) Give-a-Number task. A small basket with 15 identical bottle caps was presented to children. The task was introduced as a role-play game in which the experimenter played the role of a customer, children played the role of the grocer, and the bottle caps were supposed to be oranges. The experimenter said, "Let's play the market game! You are a grocer and I'm a customer who wants to buy some delicious oranges. Okay? Are you ready?" The experimenter then said, "Hello! May I have X orange/s, please?" As soon as children gave the selected number of oranges, the experimenter said, "Is this/Are these X orange/s?" Children were allowed to modify the number of oranges until they were sure about the number. The experimenter always started by asking for 2 oranges, and then 1, 3, 4, 5, 8, and 10 oranges were requested in a random order. Each numerosity was asked twice with a brief pause between sessions.

Aim the target task

A shooting target (red center with black–white–black concentric layers) was presented halfway from the top of the screen. Children were requested to click on the center of the target by moving the cross-shaped mouse cursor and pressing one of the mouse buttons. After clicking, a new shooting target appeared in a new randomly selected location. A total of 16 targets were sequentially presented in 16 randomly selected equidistant (71 pixels) locations starting from 71 pixels from the left edge of the computer screen. The movements of the mouse were restricted to the horizontal line, and for each trial the absolute distance in pixels between the location selected by children and the target center was calculated. For each participant, we computed the median absolute distance as an index of precision in controlling the mouse cursor. We administered this task to assess children's precision in moving the mouse cursor.

Numerosity estimation

Children were asked to verbally estimate the numerosity of briefly presented numerical sets. Before starting each trial, the experimenter waited for children to be focused on the center of the computer screen and ready for the upcoming trial. A numerical set composed of black squares was presented for 1 s to prevent serial counting (Reeve, Reynolds, Humberstone, & Butterworth, 2012) and immediately replaced by a scrambled image for 200 ms. Children's verbal estimate was recorded by the experimenter before starting the next trial. In case of hesitation, children were always invited to provide an estimate. There were two practice trials (i.e., 4 and 10) to familiarize children with the task, but feedback was not provided. Then, the numerosities 4, 6, 8, and 10 were randomly displayed four times for a total of 16 trials. For each target numerosity, item size was constant in half of the sets (i.e., size-constant condition), whereas it diminished with increasing numerosity in the other half (i.e., size-diminished condition) (equal cumulative surface area). Each target numerosity was randomly selected from a pool of 20 images, 10 for the size-constant condition and 10 for the size-diminished condition. The images had a size of 434×434 pixels, and the size of a square was 50×50 pixels in the size-constant condition. For each participant, we calculated the mean absolute deviation between the estimate and the target number.

DOS task

In the DOS task, children were required to spatially arrange two sequentially presented digits. The task began with the presentation of a black horizontal line in the middle of the entire screen and a digit (hereafter centered digit; height = 1.4 cm) placed below the middle point of the line (see Fig. 1). As soon as children moved the mouse, another digit (hereafter Target 1) appeared above the line. Target 1 appeared at only two possible locations, one left and one right, equidistant (i.e., 128 pixels) from the centered digit. Children moved Target 1 to the selected position relative to the centered digit using the mouse and then clicked one of the mouse buttons to place the number onto the line. Specifically, the experimenter said, "If the number X [pointing at centered digit] is here, where should the number Y [pointing at Target 1] be placed?" Target 1 was always one unit smaller or larger compared with the centered digit and could only be placed in one of the two possible locations, one on the left and one on the right, equidistant from the centered digit. Therefore, the position of Target 1 determined the direction of the mapping, either left to right (e.g., 1-2) or right to left (e.g., 2-1). When children clicked one of the mouse buttons, Target 1 appeared in the selected location below the line. Once Target 1 was placed, the experimenter asked children to confirm that they were sure about their decision, and if they were not they were provided with the opportunity to change the position of the target.

Following this, another digit (hereafter Target 2) appeared above the centered digit, and children were requested to place it on the line. The experimenter said, "If the number X [pointing at centered digit] is here and you placed the number Y [pointing at Target 1] here, where should the number Z

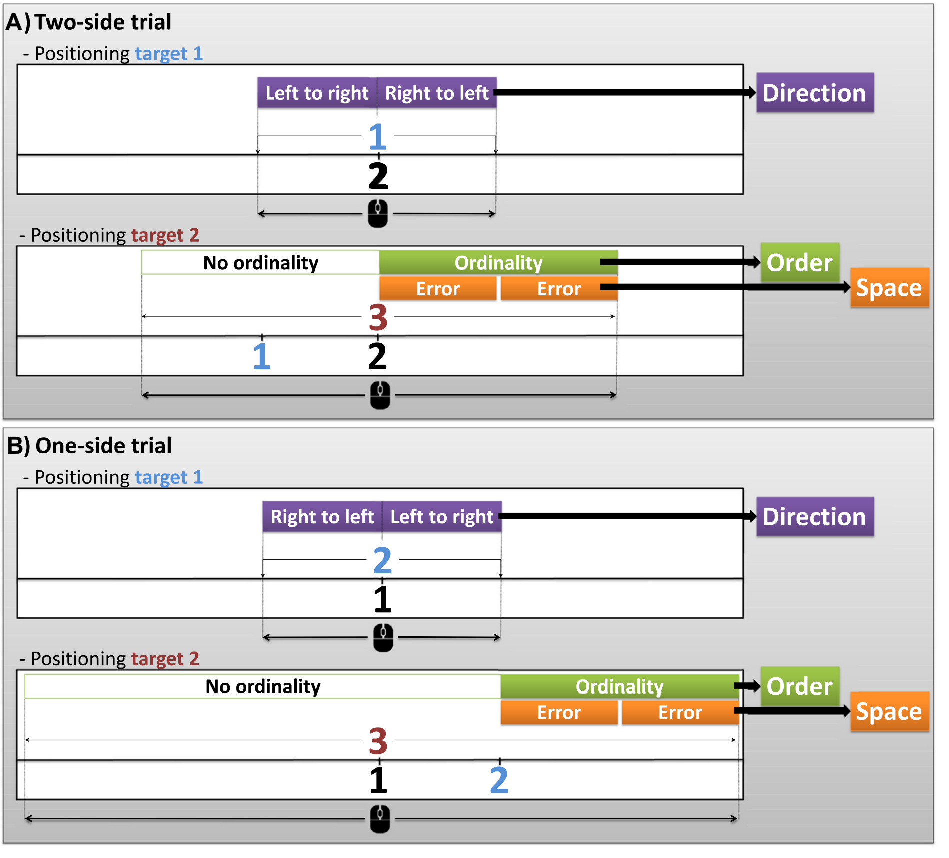

Fig. 1. (A) An example of two-side trial for the triplet 1-2-3. The number 2 was presented in the center of the line (i.e., centered digit). Then, the child moved the mouse leftward or rightward to place the number 1 (i.e., Target 1) in one of the two fixed locations. Thereafter, the number 3 (i.e., Target 2) appeared above the centered digit and the child needed to find its correct location on the line. The positioning of Target 2 determined whether the triplet of digits was correctly ordered. The movement of the mouse cursor was constrained on the line in order to have the maximum absolute error equal to 1 (i.e., estimating one unit more or less). After each positioning, the child was requested to confirm the selected location; otherwise, the child could place the target digit again. The presented digits all were in black font. (B) Example of one-side trial for the triplet 1-2-3. The structure was identical to that of the two-side trials. In the one-side trials, Targets 1 and 2 both were to be placed on the left or right side with respect to the centered digit to achieve a correctly ordered triplet.

[pointing at Target 2] be placed?” Target 2 was always one unit more or less compared with the centered digit or Target 1, depending on the type of trial (see below). Target 2 could be moved along the line using the mouse cursor, whose movement was restrained so that the maximum under- or overestimation was one unit when the spatial order of the three digits was respected (with the exception of the case in which children attempted to place Target 2 on the location of Target 1 or on the location of the centered digit). However, the mouse could nonetheless be moved by the same extent on the opposite side of the line even if locating Target 2 in that segment did not respect ordinality. When children clicked one of the mouse buttons, Target 2 appeared in the corresponding location below the line. Once children positioned Target 2, the experimenter once again asked children whether they were sure about their decision. If children placed target digits exactly on the location of the centered digit or on Target 1, a warning message appeared informing children that the position was already occupied and inviting them to find another location. The movements of the mouse were restricted when placing Target 2 to balance the possible amount of under- and overestimation. For example, in the case of the triplet 1-2-3, the distance between 2 (i.e., the centered digit) and 1 (i.e., the first target) is fixed (i.e., 1---2). Then, children are asked to place the number 3 (i.e., the second target). If children underestimate the position of 3, they would place it next to 2 (i.e., 1---23). Therefore, if the order is respected, the maximum amount of underestimation is one unit. Conversely, if children overestimate the position of 3, they would place it so that the distance between 2 and 3 is larger compared with the distance between 1 and 2 (i.e., 1---2------3). The movements of the mouse were restricted so that the maximum amount of overestimation was one unit (i.e., placing number 3 on the position of number 4).

There were seven triplets—1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, and 7-8-9—whose digits were presented in four different orders (e.g., 2-1-3, 2-3-1, 1-2-3, and 3-2-1) twice for a total of 56 trials. Half of the trials had a two-side arrangement, whereas the other half had a one-side arrangement (Table 1); in two-side trials the target digits should be placed with one on the left and one on the right side compared with the centered digit, whereas in one-side trials both target digits should be placed on the left or right side of the centered digit.

The magnitude of Target 2 could be either smaller or larger compared with the centered digit or Target 1. There were two training trials (i.e., 2-1-3 and 2-3-4) repeated twice to familiarize children with the task, and no feedback was provided. For each trial, we recorded whether the order was respected or not regardless of directionality. In case of respected order, directionality (i.e., left to right or right to left) was recorded and the absolute error between Target 2 and the estimate was calculated. Given the constraint to mouse movements, the absolute error could vary between 0 and 1. A video showing examples of a one-side trial and a two-side trial is provided at https://osf.io/nm2bz/?view_only=b5e02fe8b66c48e0952dc0608d43c7ae.

Digit comparison

For each trial, three digits (height = 2.4 cm) were presented on the computer screen at locations corresponding to 20%, 50%, and 80% of the horizontal screen size. The presentation of triplets allowed

Table 1
Configuration of trials in the DOS task.

Triplet	Centered	Target 1	Target 2	Type	Target 2 relative magnitude
1-2-3	2	1	3	Two-side	Large
1-2-3	2	3	1	Two-side	Small
1-2-3	1	2	3	One-side	Large
1-2-3	3	2	1	One-side	Small
2-3-4	3	2	4	Two-side	Large
2-3-4	3	4	2	Two-side	Small
2-3-4	2	3	4	One-side	Large
2-3-4	4	3	2	One-side	Small
3-4-5	4	3	5	Two-side	Large
3-4-5	4	5	3	Two-side	Small
3-4-5	3	4	5	One-side	Large
3-4-5	5	4	3	One-side	Small
4-5-6	5	4	6	Two-side	Large
4-5-6	5	6	4	Two-side	Small
4-5-6	4	5	6	One-side	Large
4-5-6	6	5	4	One-side	Small
5-6-7	6	5	7	Two-side	Large
5-6-7	6	7	5	Two-side	Small
5-6-7	5	6	7	One-side	Large
5-6-7	7	6	5	One-side	Small
6-7-8	7	6	8	Two-side	Large
6-7-8	7	8	6	Two-side	Small
6-7-8	6	7	8	One-side	Large
6-7-8	8	7	6	One-side	Small
7-8-9	8	7	9	Two-side	Large
7-8-9	8	9	7	Two-side	Small
7-8-9	7	8	9	One-side	Large
7-8-9	9	8	7	One-side	Small

a precise exploration of the knowledge of number magnitude of the entire range between 1 and 9 as well as a direct comparison with the triplets in the DOS task. At the beginning of each trial, the experimenter named aloud the three digits from left to right and simultaneously pointed at digits being read out. Children were requested to point at or name aloud the largest digit, and then the experimenter recorded the response. There were seven numerical triplets: 1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, and 7-8-9. The location of the three digits varied according to the six possible permutations (e.g., 1-2-3, 2-3-1, 1-3-2, 3-1-2, 3-2-1, and 2-1-3) for a total of 42 trials. For each trial, we calculated the response deviation in terms of difference between the largest digit (i.e., correct answer) and the provided response. Thereafter, we computed the mean response deviation for each participant.

Procedure

Children were met individually in a quiet room during school hours. Each session started with the simple dot comparison, forward enumeration, naming, GaN, and aim the target tasks. Thereafter, the digit comparison, DOS. and numerosity estimation tasks were presented in counterbalanced order. Children were allowed to take a break between tasks or interrupt the session without receiving any penalization. Computerized tasks were programmed using E-Prime 2.0 software (Psychology Software Tools, 2012) and were administered on a laptop (15.6-inch screen, resolution 1366 × 768 pixels; for the DOS and aim the target tasks, the resolution was set to 1280 × 768 pixels).

Results

Statistical analyses were conducted using the free software R (R Development Core Team, 2013) along with the BayesFactor package (Morey & Rouder, 2015) for Bayesian analyses using default priors. For Bayesian *t* tests we reported Bayes factors (BF_{10}) expressing the probability of the data given H1 relative to H0 (i.e., values larger than 1 are in favor of H1, and values smaller than 1 are in favor of H0), whereas for Bayesian regression analyses we reported the Bayes factors as the ratio of BF_{10} values between compared models. If the ratio between BF_{10} of Model A and BF_{10} of Model B is larger than 1, then there is evidence for Model A. Conversely, if the ratio is smaller than 1, then there is evidence for Model B. We described the evidence associated with BFs as "anecdotal" ($1/3 < BF < 3$), "moderate" ($BF < 1/3$ or $BF > 3$), "strong" ($BF < 1/10$ or $BF > 10$), "very strong" ($BF < 1/30$ or $BF > 30$), or "extreme" ($BF < 1/100$ or $BF > 100$) (Jeffreys, 1961). The raw data can be found at https://osf.io/nm2bz/?view_only=b5e02fe8b66c48e0952dc0608d43c7ae.

Simple dot comparison, forward enumeration, naming, GaN, and aim the target tasks

All participants responded correctly to the items of the simple dot comparison task except for 1 child who committed an error. In the forward enumeration task, the mean of the maximum correctly recited number was 23 (*SD* = 14). Importantly, all children recited the numbers up to 9 without committing any mistake. The percentage of correct responses in the naming task was 89% (*SD* = 21, range = 0–100), with a relevant variability induced by 4 children who correctly named less than half of the digits. We analyzed performance in the GaN task using a Bayesian model developed by Negen, Sarnecka, and Lee (2012) in order to determine the knower level for each child. The individual knower level corresponded to the highest peak of the child's posterior distribution provided by the Bayesian model assuming the same prior probability for each knower level. All children were classified as CP-knowers except for 1 child who was classified as a 4-knower and, therefore, was removed from the sample. The mean of individual median absolute distance between the presented targets and the selected locations in the aim the target task was only 2.52 pixels (*SD* = 1.23, range = 1–6).

In summary, preschool children involved in the current study displayed an understanding of "more numerous," the mastering of the cardinality principle, the ability to enumerate numbers up to at least 9, and a precise use of the mouse. The ability to correctly name Arabic digits was high but not at ceiling. However, the digits presented in the DOS and digit comparison tasks were always read by the experimenter.

Numerosity estimation

The provided estimates ranged from 1 to 21, and all were included in the following analysis. We analyzed the mean absolute difference in a repeated-measures Bayesian analysis of variance (ANOVA) with target numerosities (4, 6, 8, or 10) and type of trial (size diminished or size constant) as within-participant factors. The model with only the main effect of target numerosities yielded the highest evidence ($BF_{10} = 2.44 \times 10^{29}$). For each participant, we ran a linear regression with the mean estimates as a function of target numerosities to verify whether children provided estimates that linearly increased with target numerosities. The mean of individual beta coefficients was positive and different from zero ($M = 0.77$, $SD = 0.36$, 95% confidence interval (CI) [0.67, 0.86], $BF_{10} = 1.76 \times 10^{21}$ extreme evidence), suggesting an increase in estimates with larger target sets. We also calculated the individual regression slopes of the estimates as a function of target numerosities from 6 to 10 ($M = 0.68$, $SD = 0.47$, 95% CI [0.56, 0.80]). According to Le Corre and Carey (2007), children with a slope ≥ 0.3 were classified as mappers, whereas children with a lower slope were classified as non-mappers. In our sample, 52 children were classified as mappers and 10 were classified as non-mappers. It follows that most of the children knew that larger numerical quantities are associated with later number words in the counting list (i.e., later-greater principle).

DOS task

We calculated the proportion of trials in which the three presented digits were correctly ordered (i.e., ordinality). Then, for the correctly ordered trials, we computed the proportion of time the left-to-right direction was displayed and the absolute error between the provided position of Target 2 and its correct location on the line (Fig. 2). For the analyses of the directionality and the absolute error of mapping, 4 participants (all CP-mappers) were excluded because they failed to correctly order at least one trial for all the triplets.

We first verified whether the accuracy in ordering digits varied among the triplets of digits. We ran a Bayesian repeated-measures ANOVA with mean proportion of correctly ordered trials as the dependent variable and triplet (1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, or 7-8-9) as a within-participant factor. There was extreme evidence for the model with the main effect of triplet ($BF_{10} = 36909$). For each participant, we ran a linear regression with the mean proportion of correctly ordered trials as a function of triplet magnitude (denoted by numbers from 1 to 7). The mean beta coefficient was negative, and there was extreme evidence for its difference from zero ($M = -0.02$, $SD = 0.04$, 95% CI [-0.03, -0.01], $BF_{10} = 181$), suggesting a decrease in ordering accuracy with larger triplets of digits.

We also assessed whether the use of the left-to-right mapping varied among the triplets of digits. We ran a Bayesian repeated-measures ANOVA with the mean proportion of left-to-right direction of mapping as the dependent variable and triplet (1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, or 7-8-9) as a within-participant factor. There was moderate evidence against the effect of triplet ($BF_{10} = 0.13$),

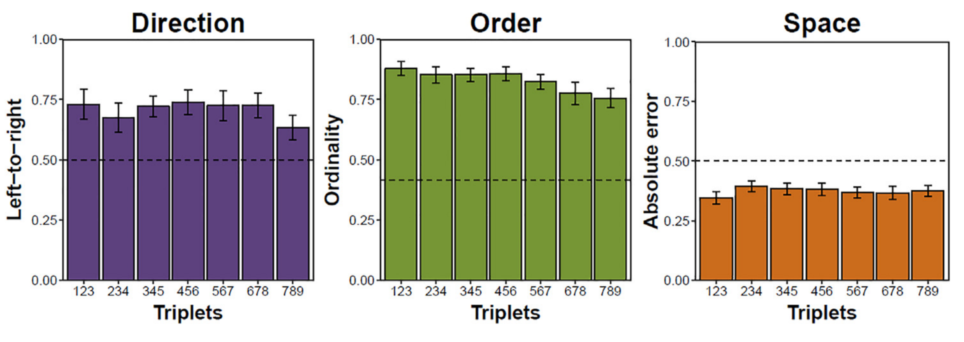

Fig. 2. Direction: Proportion of left-to-right mapping for correctly ordered trials separately for each triplet. Order: Proportion of correctly ordered trials separately each triplet. Space: Proportion of absolute error of spatial mapping for correctly ordered trials separately for each triplet. Error bars represent within-participant 95% confidence intervals. Dashed lines represent the chance levels.

Table 2
Distribution of preferred direction as a function of ordinality being above or at the chance level and the presence of a defined or undefined mapping.

Preferred direction	Above chance-level ordinality		At chance-level ordinality	
	Defined mapping	Undefined mapping	Defined mapping	Undefined mapping
Left to right	37	5	1	2
Right to left	10	5	0	0
Equiprobable	–	1	–	1

Note. Cell values represent numbers of children ($N = 62$).

suggesting stable use of the same mapping direction for all the triplets. To investigate the directionality at the individual level, children were classified according to their performance being above or at the chance level for ordinality and their mapping being defined or undefined. The mapping was considered as defined when the number of trials with left-to-right direction out of the number of correctly ordered trials was significantly above (i.e., preferred direction from left to right) or below (i.e., preferred direction from right to left) the chance level (binomial test). For children who were above the chance level in terms of ordinality and with a defined mapping, the left-to-right mapping was predominant (37 children) compared with the right-to-left mapping (10 children) (Table 2). Children who used the left-to-right mapping and right-to-left mapping the same number of times were classified as having an equiprobable mapping direction.

Finally, we verified whether the error in placing digits varied among the triplets of digits. We ran a Bayesian repeated-measures ANOVA with the mean absolute error as the dependent variable and triplet (1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, or 7-8-9) as a within-participant factor. There was moderate evidence for the exclusion of the effect of triplet ($BF_{10} = 0.11$), suggesting stable accuracy in placing target digits across triplets.

There was moderate evidence for a positive correlation between the mean proportion of correctly ordered triplets (i.e., ordinality) and the mean proportion of left-to-right mapping (i.e., direction), $r(60) = .35$, $BF_{10} = 7.37$, whereas there was anecdotal evidence for a lack of correlation between ordinality and absolute error, $r(60) = -.15$, $BF_{10} = 0.45$, and between direction and absolute error, $r(60) = -.15$, $BF_{10} = 0.47$.

Digit comparison

We assessed whether the response deviation in the digit comparison task varied among the triplets of digits. We analyzed the mean response deviation from the digit comparison task in a Bayesian repeated-measures ANOVA with triplet (1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, or 7-8-9) as a within-participant factor (Fig. 3). There was very strong evidence for the main effect of triplet ($BF_{10} = 87$). For each participant, we ran a linear regression with the mean response deviation as a function of triplet magnitude (from 1 to 7). The mean beta coefficient was positive and different from zero ($M = 0.03$, $SD = 0.08$, 95% CI [0.01, 0.05], $BF_{10} = 7.87$, moderate evidence), suggesting a marginal increase in response deviation (i.e., worse performance) in larger triplets of digits.

Regression analyses

We ran a Bayesian regression analysis to investigate the relation between different numerical skills and number comparison performance (Table 3). In the regression model, we inserted the mean proportion of correctly ordered triplets (i.e., ordinality), the mean proportion of left-to-right mapping (i.e., direction), the mean absolute error from the DOS task, the mean absolute difference in the numerosity estimation task, and age in months as predictors of the mean response deviation in the digit comparison task.[1] The model including ordinality and the mean absolute difference in the

[1] The pattern of results of the regression analyses reported in Table 3 remained unchanged when we used accuracy in the digit comparison task as an outcome variable.

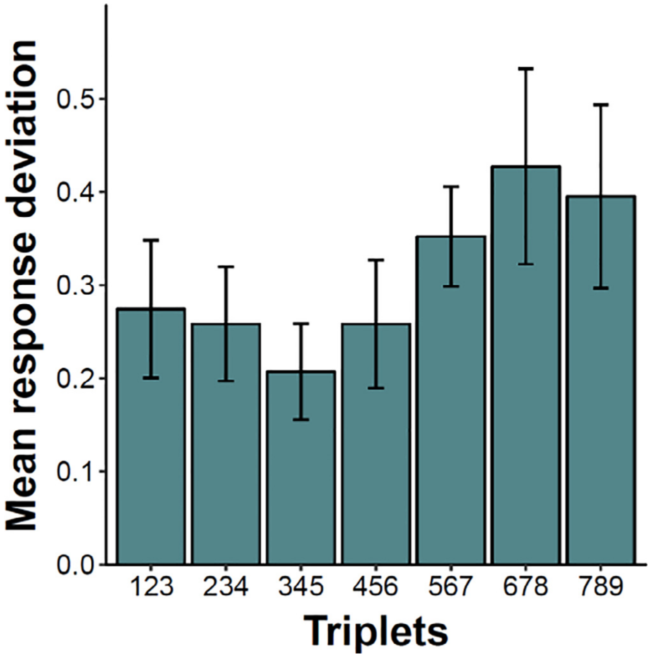

Fig. 3. The response deviation as a function of triplets in the digit comparison task. Error bars represent 95% confidence intervals.

Table 3
Summary of the Bayesian regression models with mean response deviation in the digit comparison task as outcome variable.

Model	Measure	*B*	95% Confidence interval	BF_{10}	R^2	Model comparison
1	Ordinality[a]	−0.92	[−1.31, −0.53]	40,034	.39	–
	Numerosity estimation[b]	0.11	[0.02, 0.21]			
2	Ordinality[a]	−1.06	[−1.44, −0.67]	15,945	.34	BF_{10} Model 1/BF_{10} Model 2 = 2.51
3	Numerosity estimation[b]	0.18	[0.07, 0.28]	26.65	.16	BF_{10} Model 1/BF_{10} Model 3 = 1502

[a] Ordinality is the mean proportion of correctly ordered trials.
[b] Numerosity estimation is the mean absolute difference between target numerosities and estimates.

numerosity estimation task yielded the largest evidence (Model 1: BF_{10} = 40034). The main assumptions of linear regression were respected in Model 1 (Peña & Slate, 2006). However, there was anecdotal evidence (BF = 2.51) for Model 1 when compared with the model including only ordinality (Model 2: BF_{10} = 15945). Conversely, there was extreme evidence (BF = 1502) for Model 1 when compared with the model including only the mean absolute difference in the numerosity estimation task (Model 3: BF_{10} = 27). In summary, there was extreme evidence for ordinality as a predictor of the performance in the digit comparison task, whereas there was anecdotal evidence for numerosity estimation.

The pattern of results remained stable when the mean absolute error in the DOS task was replaced by the residuals of this measure obtained removing the precision in moving the mouse cursor (i.e., median absolute deviation from the aim of the target task). Moreover, the model with only ordinality also obtained the largest evidence (BF_{10} = 15945) when the absolute difference in the numerosity estimation task was replaced with a variable coding for the status of mapper (=1) and non-mapper (=0) as proposed by Le Corre and Carey (2007). Finally, in previous studies (Davidson et al., 2012; Le Corre, 2014), numerosity estimation was specifically related to the comparison of large (i.e., >4) number words given that small number words can conceivably be mapped to the corresponding numerical quantities via the OTS. Therefore, we ran the same regression models using the mean difference of the digit comparison task only for large numbers (from 4-5-6 to 7-8-9) as an outcome variable, and we also calculated the ordinality, direction, and error only from triplets from 4-5-6 to 7-8-9. Again, the model with only ordinality displayed the largest evidence (BF_{10} = 176631).

Spatial mapping of correctly ordered digits

We further investigated the spatial mapping of digits in the DOS task by exploring the positioning of Target 2 in correctly ordered trials. In Fig. 4, we report the distribution of estimated Target 2 positions for each triplet and separately for trials in which Target 2 was smaller or larger compared with Target 1 or the centered digit.

In correctly ordered trials, the position of Target 2 could vary between −1 and +1 units with respect to the correct position on the line (i.e., zero error). When the relative magnitude of Target 2 was small (compared with Target 1 or the centered digit), a positive error value represents underestimation, whereas a negative value represents overestimation. Conversely, when the relative magnitude of Target 2 was large, a positive value represents overestimation and a negative value represents underestimation. Crossing under- versus overestimation of the position of Target 2 with relative magnitude (small vs. large) leads to four different types of mapping (Fig. 5): underestimation (where Target 2 is systematically underestimated regardless of its relative magnitude), overestimation (where Target 2 is systematically overestimated regardless of its relative magnitude), compression (where Target 2 is overestimated when its relative magnitude is small, whereas it is underestimated when its relative magnitude is large), and expansion (where Target 2 is underestimated when its relative magnitude is small, whereas it is overestimated when its relative magnitude is large).

For each child, we calculated the mean and 95% CIs of the positioning of Target 2 separately for the trials in which Target 2 was smaller or larger compared with Target 1 or the centered digit. A type of

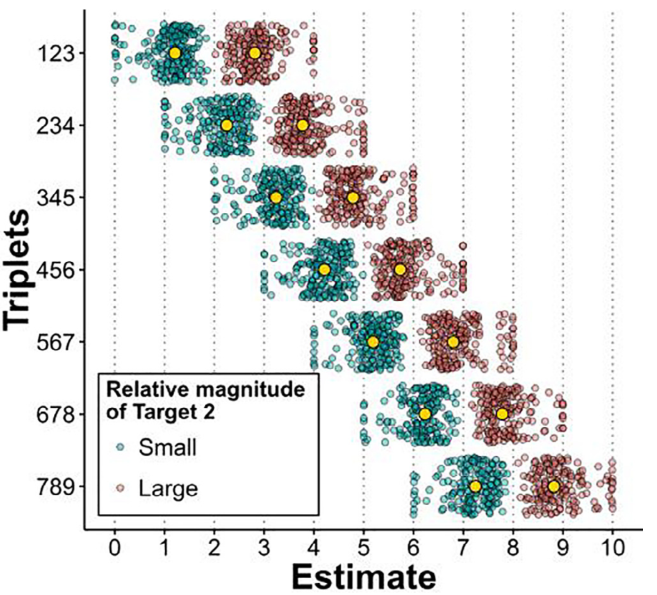

Fig. 4. Distribution of estimates in correctly ordered trials from the DOS task for each triplet. Green dots represent the estimates when Target 2 was smaller compared with the centered digit or Target 1. Red dots represent the estimates when Target 2 was larger compared with the centered digit or Target 1. Yellow dots represent the median of the distributions. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

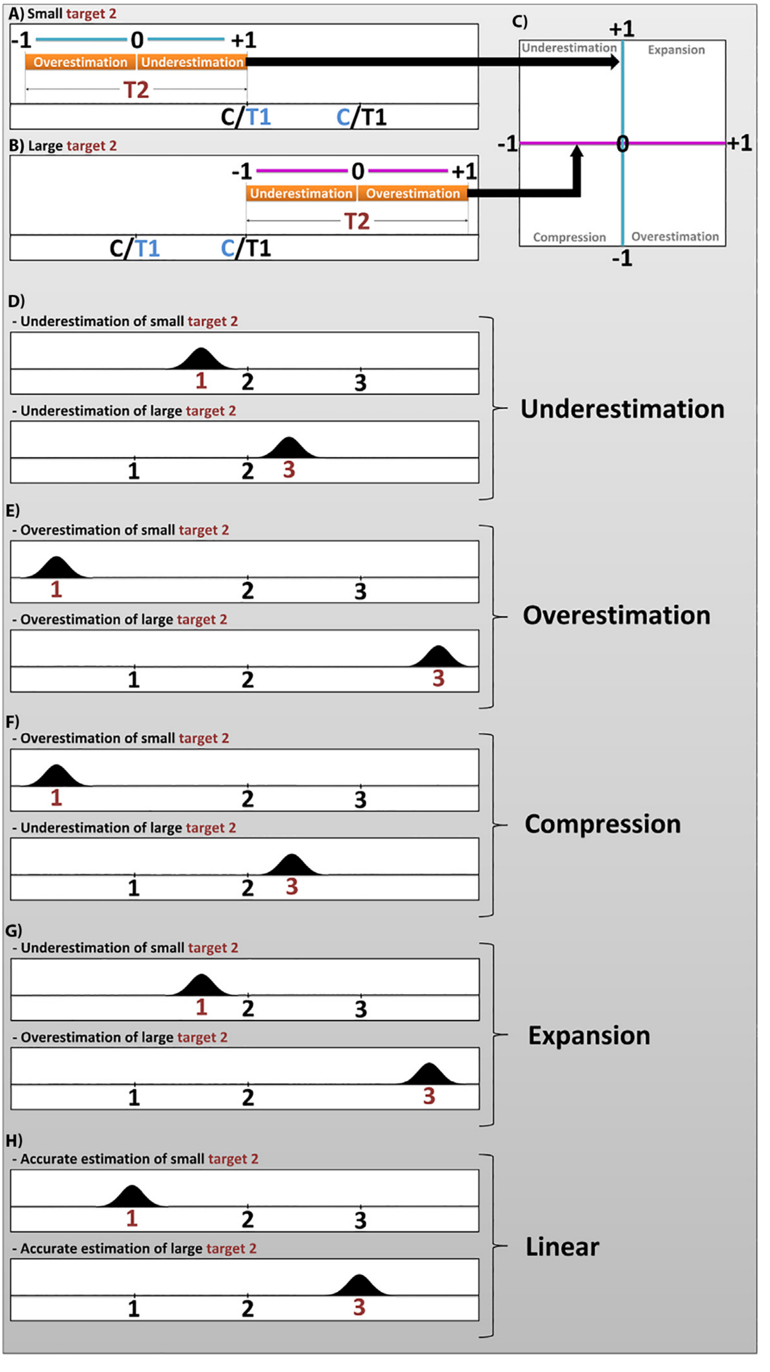

Fig. 5. (A) Schematic representation of the positioning of Target 2 when it was smaller compared with Target 1 or the centered digit. The positioning of Target 2 could vary between −1 and +1 unit. (B) Schematic representation of the positioning of Target 2 when it was larger compared with Target 1 or the centered digit. (C) Positioning of Target 2 when it was smaller (*y* axis) or larger (*x* axis) compared with Target 1 or the centered digit represented on Cartesian axes. The four quadrants represent different patterns of spatial mapping for correctly ordered triplets of digits. (D–H) Positioning of Target 2 according to different types of mapping.

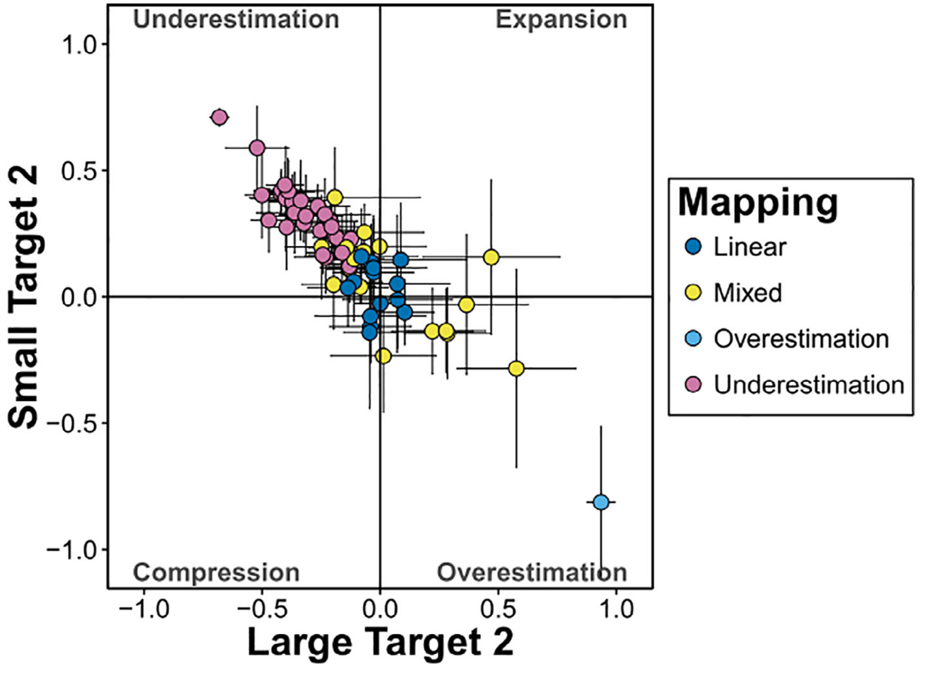

Fig. 6. Colored points represent individual means of Target 2 positioning for trials in which Target 2 was smaller (*y* axis) or larger (*x* axis) compared with Target 1 or the centered digit. Error bars represent 95% confidence intervals. (For interpretation of the reference to color in this figure legend and the color displays in the figure key, the reader is referred to the Web version of this article.)

mapping was assigned to triplets and children according to the location within the four quadrants. Nevertheless, when one of the 95% CIs overlapped one of the Cartesian axes, the mapping was considered as mixed between two types of mapping. When both the 95% CIs overlapped the two Cartesian axes, the mapping was considered as *linear.*

At the individual level, 30 participants displayed a clear pattern of underestimation, 17 displayed a mixed pattern, 14 were classified as linear, and only 1 displayed a pattern of overestimation (Fig. 6). Despite the capability of the DOS task to detect biased loglike mapping as observed in the NL task (see online supplementary material), none of the children in our sample displayed clear compressed mapping.

Discussion

In this study, we investigated how direction, order, and accuracy of spatial mapping relate to the understanding of magnitude relations among symbolic numbers in preschool children. The ability to compare numbers was associated with spatial order, whereas there was no evidence for association with direction or accuracy of spatial mapping. These results confirm the intimate link between space and symbolic magnitude representation in young children (Sella et al., 2017). Here we significantly extended previous findings by highlighting that spatial ordering of numbers is a crucial component for the development of number comparison skills. This fits well with the recent proposal that ordinality is a key aspect for the development of symbolic mathematical knowledge (De Visscher, Szmalec, Van Der Linden, & Noël, 2015; Lyons & Ansari, 2015; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Vogel, Remark, & Ansari, 2014; Vos, Sasanguie, Gevers, & Reynvoet, 2017). We acknowledge that the correlational nature of the findings prevents any causal inference regarding the association between spatial mapping and symbolic number comparison. Nevertheless, training based on the spatial representation of numbers (e.g., linear board games) has been demonstrated to improve digit comparison skills in preschool children (Ramani, Siegler, & Hitti, 2012; Siegler & Ramani, 2009). Moreover, training preschool children on the sequential order of digits (i.e., individuating the predecessor and successor of a given digit) resulted in improved ability in ordering digits according to their magnitude and in better performance in the NL task with the 0–10 interval (Xu & LeFevre, 2016). In adults, learning the numerical magnitude of new artificial symbols was found to be more efficient when symbols were associated with horizontally arranged spatial locations (i.e., magnitude increasing from left to right) than with sets of dots (i.e., via the ANS) (Merkley, Shimi, & Scerif, 2016). Taken together, these results indicate that space can be an efficient and primary structure to represent symbolic numerical magnitude (Sella et al., 2017, see also Zorzi et al., 2012, for converging evidence from neuropsychology).

In line with previous studies, we found that both numerosity estimation and spatial mapping were related to symbolic comparison skills in young children (Le Corre, 2014; Sella et al., 2017). Nevertheless, there was convincing evidence in favor of spatial order, whereas there was no decisive evidence for numerosity estimation. This discrepancy may relate to the structure of the digit comparison task; numerals were visually presented and remained on the computer screen until response, whereas the corresponding number words read aloud by the experimenter were only briefly available to children. Predominance of the visual format might have promoted the use of spatial knowledge (Sella et al., 2017, Sella et al., 2018). Conversely, when presented with only with number words, children might be more prone to use verbal strategies such as counting and the later-greater principle (Le Corre, 2014; see Benoit, Lehalle, Molina, Tijus, & Jouen, 2013; Hurst, Anderson, & Cordes, 2016; Jiménez Lira, Carver, Douglas, & LeFevre, 2017; and Knudsen, Fischer, Henning, & Aschersleben, 2015, for studies investigating the mapping between different numerical formats in young children).

Preschool children in our sample already displayed a preference for the left-to-right spatial arrangement of numbers even though the non-canonical right-to-left mapping was also observed, thereby confirming that the association between numbers and space during the early stage of development is flexible (McCrink & Opfer, 2014; Opfer, Thompson, & Furlong, 2010; Patro, Fischer, Nuerk, & Cress, 2016). Children with a better performance in ordering triplets were also more likely to place digits with increasing numerical magnitude from left to right. This association may stem from the fact that children were learning the spatial arrangement of digits along the number line according to the culturally determined left-to-right direction (Shaki, Fischer, & Göbel, 2012). However, the use of the left-to-right mapping was not associated with better ability in comparing Arabic numbers.

Several conflicting theoretical accounts have been proposed to explain the shift from biased (log-like) to accurate (linear) mapping in the NL task, thereby giving rise to a lively debate (Barth & Paladino, 2011; Dackermann, Huber, Bahnmueller, Nuerk, & Moeller, 2015; Opfer et al., 2011). Most of these studies have implemented and compared a variety of psychophysical functions (e.g., power, logarithmic, bilinear) to determine the best fit to the pattern of estimates from the NL task. However, statistical modeling has emphasized descriptive accounts specifically related to the NL task rather than better characterize the cognitive operations related to the spatial mapping of numbers. In contrast, the novel task developed in the current study allowed us to simultaneously and separately assess direction, order, and spacing components of spatial mapping. In correctly ordered trials, the positioning of target digits revealed the presence of three main types of mapping: underestimation, mixed, and linear. Most of the biased mappings resulted in a pattern of underestimation; that is, Target 2 was placed slightly closer to Target 1 or the centered digit than the expected location based on equal spacing. We speculate that once the order of digits was respected, children moved Target 2 toward its correct location (i.e., error equals zero) but tended to place it before getting to the exact position on the line. Despite our data supporting a pattern of underestimation, we cannot exclude the presence of compressed mapping with larger numerical intervals in which the effect of compression might be more evident.

The finding that proficient digit comparison is linked to the emergence of consistent spatial mapping of numbers suggests that the encoding of space-based ordinal relations is an important facet of exact number representation, which has been largely neglected by mainstream developmental theories that emphasize the foundational role of one or both of the preverbal number systems (Carey & Xu, 2001; Feigenson et al., 2004; Piazza, 2010; but see Reynvoet & Sasanguie, 2016). In our daily life environment, digits are frequently represented using a spatial layout with numerical magnitude increasing in a specific direction (usually from left to right in Western cultures but also from bottom to top). Although the origins of the classic horizontal arrangement may reside in situated and culture-dependent aspects of cognition (Fischer, 2012; see also Blini, Pitteri, & Zorzi, 2018, for a discussion in relation to mental arithmetic), an external spatial layout can act as a powerful source of information because the numerical magnitude of a digit can be conveyed by its relative location compared with other digits. Children who have interiorized the spatial arrangement of digits along the line can use this information to derive the numerical meaning of numerals. We speculate that efficient visuospatial processing can strengthen the association between numbers and space, thereby fostering the distinctiveness of each digit within the number line (Rinaldi, Gallucci, & Girelli, 2016). This scenario would fit with the results from other studies that revealed a link between visuospatial skills and performance in the NL task (Gunderson, Ramirez, Beilock, & Levine, 2012; Sella, Sader, Lolliot, & Cohen Kadosh, 2016; Simms, Clayton, Cragg, Gilmore, & Johnson, 2016; Thompson, Nuerk, Moeller, & Cohen Kadosh, 2013). Another way in which numbers are linked to space is through counting activities. In fact, children spontaneously map the increments in numerical magnitude on space when pointing to counted objects according to a preferred counting direction (Rinaldi, Gallucci, et al., 2016), which may explain the presence of some children displaying a mapping from right to left in the current study. In this scenario, body movements (i.e., fingers, arm) accompanying counting may support the acquisition of numerical magnitude associated with numerals (Rinaldi, di Luca, Henik, & Girelli, 2016; embodied cognition: Wilson, 2002). Nevertheless, the observed relation between ordinality and symbolic magnitude may be originally non-spatial. Children who can efficiently access number positions in the counting list can also spatially reorder them using a verbal strategy (e.g., "Seven should be placed here because it comes after five and six"). Future studies are needed to clarify the contribution of verbal and spatial order to the understanding of magnitude relation between number words and numerals.

Conclusions

We designed a new spatial mapping task to investigate direction, ordinality and accuracy of spatial mapping of digits in preschool children. Ordinality knowledge tended to decrease for the larger digits, and the direction of the spatial mapping was predominantly from left to right. In correctly ordered trials, children tended to place digits before getting to the correct location on the line, thereby displaying a generalized pattern of underestimation with no evidence for a compressed number line. Crucially, only ordinality was correlated with digit comparison performance, whereas direction and accuracy of mapping were not. Our results suggest that the spatial order of digits can act as a powerful source of magnitude information that young children use to scaffold the mental representation of exact numbers.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jecp.2018.09.001.

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