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Representation of numerical and non-numerical order in children

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ABSTRACT

The representation of numerical and non-numerical ordered sequences was investigated in children from preschool to grade 3. The child's conception of how sequence items map onto a spatial scale was tested using the Number-to-Position task (Siegler & Opfer, 2003) and new variants of the task designed to probe the representation of the alphabet (i.e., letter sequence) and the calendar year (i.e., month sequence). The representation of non-numerical order showed the same developmental pattern previously observed for numerical representation, with a logarithmic mapping in the youngest children and a shift to linear mapping in older children. Although the individual ability to position non-numerical items was related to the child's knowledge of the sequence, a significant amount of unique variance was explained by her type of number-line representation. These results suggest that the child's conception of numerical order is generalized to non-numerical sequences and that the concept of linearity is acquired in the numerical domain first and progressively extended to all ordinal sequences.

1. Introduction

How numerical information is represented in mind and brain, as well as the developmental pathway that leads to adult numeracy, is a major issue in numerical cognition research. Considerable progress has been made towards understanding how animals and humans represent numerosity that is, the cardinality of a set. However, numbers can have other meanings, such as order or rank.

Ordinal information is not a distinctive property of numbers because it is shared with a variety of non-numerical sequences that we learn in a conventional fixed order during childhood. Two prominent examples are the alphabet and the months of the year. Studies that investigated the representation of ordinal information have found both similarities and differences between numbers and non-numerical ordered sequences, raising the question of whether they might depend on shared cognitive and neural substrates.

The first suggestion that numerical and non-numerical ordered sequences share common characteristics comes from the observation that comparing two items close in the sequence takes longer than two items further apart. Indeed, a graded distance effect, first reported for numbers in the seminal work of Moyer and Landauer (1967), was also found in a letter comparison task (Hamilton & Sanford, 1978; Jou & Aldridge, 1999; Van Opstal, Gevers, De Moore, & Verguts, 2008) and even after learning a new abstract ordering of elements (Woocher, Glass, & Holyoak, 1978). Van Opstal, Gevers, et al. (2008) replicated the distance effect for both number and letters in a comparison task, but they found distance-dependent priming effects only for numbers. Thus, the striking similarity between numerical and alphabetical order seems to be related to specific conditions or processes.

A second line of evidence comes from studies that investigated the spatial coding of numerical and non-numerical ordered information. The spatial–numerical association of response codes (SNARC; Dehaene, Bossini,

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& Giraux, 1993), which indexes faster left-hand than right-hand responses to small numbers and faster right-hand than left-hand responses for larger numbers (in Western cultures), has been extended to letters and months (Gevers, Reynvoet, & Fias, 2003, 2004). That is, beginning items of the sequence show an association with left responses, whereas later items are associated to right responses. This finding suggested that the representation of letters and months is spatially organized with a format that closely resembles the representation of numbers. Thus, the concept of "number-line" (Dehaene et al., 1993; Zorzi, Priftis, & Umiltà, 2002; Zorzi et al., 2012) would have a parallel for non-numerical sequences, such as an "alphabet-line" for letters. Interestingly, some individuals with synaesthesia report seeing numbers, but also non-numerical sequences (letters, months, days of the weeks), as projected onto the external visual space along a specific trajectory such as a line or a more complex graphical form (e.g., Galton, 1880; Hubbard, Ranzini, Piazza, & Dehaene, 2009; Price & Mentzoni, 2008). Nevertheless, one aspect of number processing that appears to be domain-specific is its influence on the orienting of spatial attention. Number cues cause shifts of spatial attention (leftward for small and rightward for large numbers; Fischer, Castel, Dodd, & Prett, 2003), whereas letter cues have no effect (Casarotti, Michielin, Zorzi, & Umiltà, 2007).

In a study on neglect patients, Zorzi, Priftis, Meneghello, Marenzi, and Umiltà (2006) also showed similarities and differences in the mental representations of numbers, letters and months. When asked to bisect physical lines, patients tend to misplace the center of the line to the right of the true midpoint, with a deviation that increases with line length. Zorzi and colleagues (2002) found the same spatial bias when patients were asked to mentally bisect a verbally presented numerical interval. Indeed, patients showed a systematic bias towards larger numbers (right on the mental number line), which increased as a function of the length of the interval (e.g., responding that 7 is the midpoint between 1 and 9). Zorzi et al. (2006) observed the same pattern of results for number interval bisection but not for the mental bisection of intervals formed by letters (e.g., L-P) or months (e.g., April–October). Patients showed a bias towards later items (right on a putative alphabet line) in the letter bisection task, but the bias was not modulated by interval length. For month intervals, the bisection bias was in the opposite direction that is towards the beginning items. Similarities and differences between numerical and non-numerical bisections were also observed in a subsequent study on neglect patients (Zamarian, Egger, & Delazer, 2007). Finally, studies of pseudoneglect, a general tendency of neurologically intact individuals to favor the left side of space (Manning, Halligan, & Marshall, 1990), revealed a slight "leftward" misplacement of the interval midpoint for both number bisection (Göbel, Calabria, Farnè, & Rossetti, 2006) and letter bisection (Nicholls & Loftus, 2007).

The hypothesis of a common mechanism for processing numerical and non-numerical order has also found support from neuroimaging studies (Fias, Lammertyn, Caessens, & Orban, 2007). It is widely held that the horizontal segment of the intraparietal sulcus (hIPS) plays a central role in the representation and processing of numerical information (Dehaene, Piazza, Pinel, & Cohen, 2003, for review). Notably, in a comparison task, the activation of hIPS is modulated by the numerical distance (Pinel, Dehaene, Rivière, & LeBihan, 2001). Crucially, Fias et al. (2007) showed that the hIPS is equally responsive during comparisons of numerical magnitude and letter order. Participants in their study had to judge which of two letters came later in the alphabet and the resulting activations were compared with those obtained when the task was to judge which of two numbers was larger. Highly similar neural networks were activated by number and letter comparisons, and the conjunction between number and letter comparisons showed selective activation of bilateral hIPS. Thus, hIPS activation was specifically related to order comparison, suggesting that its role might be to represent ordinality rather than just quantity/cardinality (Nieder, 2005). Further support to the hypothesis that hIPS is involved in the representation and processing of non-numerical ordered sequences is provided by the study of Ischebeck et al. (2008), who found no significant difference in IPS between ordered generation of months and numbers, compared to the generation of non-ordered names of animals. However, a recent study of Zorzi, Di Bono, and Fias (2011) has questioned the significance of the overlap in cortical activation between number comparison and letter comparison. Analyses of the fMRI data of Fias et al. (2007) using multivariate classifiers showed that number and letter comparison can be separated within hIPS even though the two tasks yielded the same metric of behavior. These results reconcile the neuroimaging data with the neuropsychological evidence suggesting dissociations between numbers and other non-numerical ordered sequences (Zorzi et al., 2006).[1]

The studies reviewed above, all concerned with adult participants, are somewhat inconclusive because they show both associations and dissociations between numerical and non-numerical order. Developmental studies might provide further insights into this issue by looking at how the different representations develop in children. Are numerical and non-numerical ordered sequences represented in a similar format? What developmental pattern leads to the acquisition of these representations? In the numerical domain, seminal studies by Siegler and Booth (2004) and Siegler and Opfer (2003) have shown that children's representation of numbers, indexed by how numbers are placed on a spatial scale (i.e., "number lines" with 0 at one end and 100 or 1000 at the other) change with age during the first years of school and shift from logarithmic to linear.[2] For example, children at grade 2 and 4 overestimated small numbers and compressed large numbers to the end of the scale (logarithmic positioning) when the context was unfamiliar (0-to-1000), but positioned numbers linearly in a familiar context (0-to-100). In contrast,

[1] Note that ordinal meaning of numbers has been shown do dissociate from cardinal meaning in single-case neuropsychological studies (Delazer & Butterworth, 1997; Turconi & Seron, 2002).

[2] A recent study of Barth and Paladino (2011) argued that the data in the Number-to-Position task are better accounted for by a proportion-judgment model. This issue is taken up in the General Discussion.

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grade 6 children positioned numbers linearly on both small and large scales, just like adults. Interestingly, younger children treated the same numbers differently – identical numbers being placed linearly or logarithmically according to the interval of reference – indicating how the context influenced the numerical representation deployed in the task and how the choice among these multiple representations is dependent of age and experience. Berteletti, Lucangeli, and Zorzi (2010) extended these findings to a younger population of preschoolers, who showed the same transition from logarithmic to linear positioning on smaller number lines (1–10 and 1–20).

To the best of our knowledge, no study to date has investigated how children represent non-numerical ordered sequences and whether it may change over development as previously shown for numbers. We therefore developed variants of Sigler and Opfer's (2003) Number-to-Position (NP) task to investigate whether non-numerical ordered sequences can be readily mapped onto lines, as it is the case for numbers, and whether the type of representation used by children shows a developmental trend resembling the pattern that has been established for numbers. As noted above, the NP task has been an effective tool for uncovering the child's mental representation of numbers as well as its developmental trajectory.

The developmental pattern with an initial logarithmic phase followed by a shift to linear positioning might be a distinctive feature of numbers, perhaps driven by their unique property of conveying cardinality. Alternatively, non-numerical ordered sequences might show the same developmental pattern of numbers, which would fit well with the similarities across domains observed in adult participants and the fact that they share the ordinal dimension. However, this would lead to the question of how the transition to linearity is related across domains. The timing of the shift might be independent across numerical and non-numerical orders; alternatively, linear positioning for non-numerical ordered sequences might be observed only once it is acquired for a comparable numerical range. The latter finding would suggest that linearity in the representation of non-numerical ordered sequences is a generalization from the numerical domain.

We examined the performance of preschoolers[3] and primary school children (grade 1, 2 and 3) in several positioning tasks. We presented two classic number lines, as previously used by Siegler and collaborators, as well as two new lines where children had to position letters and months, respectively. Thus, the Letter-to-Position (LP) task required to position letters on a line marked with "A" and "Z" at its ends, whereas the Month-to-Position (MP) task required to position months on a line marked with "January" and "December" at its ends. Moreover, to allow a direct comparison with the numerical representation, we also presented two new number lines matched to the non-numerical lines in terms of interval length (e.g., the number line matched to the month line was marked 1 and 12 at its ends). These number lines allowed us to directly test the generalization hypothesis. Indeed, if linearity for numbers is generalized to non-numerical ordinal sequences, performance on the non-numerical lines should be predicted by the performance on the corresponding number lines.

2. Method

2.1. Participants

A total of 136 children from 14 different schools of north-eastern Italy ranging from the last year of kindergarten to grade 3 took part in the study. There were 51 preschoolers (mean age = 5 yr 8 mo, *SD* = 5.4 mo, range: 58–79 mo, 27 girls), 28 children from grade 1 (mean age = 6 yr 11 mo, *SD* = 3.9 mo, range: 72–89 mo, 15 girls), 35 children from grade 2 (mean age = 7 yr 11 mo, *SD* = 3.4 mo, range: 90–104 mo, 18 girls), and 22 children from grade 3 (mean age = 8 yr 9 mo, *SD* = 4.4 mo, range: 96–116 mo, 9 girls).

2.2. Procedure

Trained teachers from each school met with the children individually during school hours in a quiet classroom for about half an hour. Order of experimental tasks was randomly presented to children. Tasks were presented as games, no time limit was given and items or questions could be repeated if asked but neither feedback nor hints were given to the child. Children were free to stop at any time.

2.2.1. Sequences knowledge

All children were tested on their minimal knowledge of numbers, letters and months of the year. Instructions for the number sequence were to count as far as they knew (e.g., "Do you know the numbers? Try to tell me all the numbers you know"), but were stopped whenever they reached 30. In the same way they were asked to say all the letters and months they knew.

2.2.2. Number-to-Position task (NP task)

Numerical estimation was assessed as in Siegler and Opfer (2003). Children were presented with 25-cm long lines in the center of a horizontal A4 sheet. Two different lines were administered: 0–100 and 0–1000. The ends of the lines were labeled on the left by 0 and on the right by either 100 or 1000. The number to be positioned was written in the upper left corner of the sheet (rather than directly above the line) to avoid its use as visual reference. Numbers to be positioned for the 0–100 line were: 2, 3, 4, 6, 18, 25, 48, 67, 71, 86; and for the 0–1000 line: 4, 6, 18, 25, 71, 86, 230, 390, 780, 810 (corresponding to sets A and B for the same lines used in Siegler & Opfer, 2003). Items were completely randomized within each interval and were presented separately from each other to avoid influence from previous positioning. Instructions given at

[3] In the Italian education system, primary school starts at 6 years of age. Children aged between 3 and 5 years can attend kindergarten but this is not compulsory and it does not imply formal teaching (indeed, classes are often mixed in terms of age groups). We will use the term preschooler throughout the article to define our youngest group of children. This term highlights the absence of formal teaching compared to the meaning of kindergarten in other countries.

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the beginning were: "We will now play a game with the number lines. Look at this page: you see there is a line drawn here. I want you to tell me where some numbers are on this line. When you have decided where the number goes, I want you to make a mark with your pencil on this line." To ensure that the child was well aware of the interval size, the experimenter would point to each item on the sheet while repeating for each item: "This line goes from 0 to 100 (1000). If here is 0 and here is 100 (1000), where would you position 50 (500)? The experimenter always named the number to be positioned.

For both intervals there was a practice trial that used numbers 50 or 500 according to the interval. Experimenters were allowed to rephrase the instructions as many times as required without making suggestions as to where to place the mark both for practice and test trials.

2.2.3. Non-numerical lines task

The Letter-to-Position (LP) task and the Month-to-Position (MP) task as well as the corresponding numerical interval lines were presented in the same way as the NP task. The stimuli used for the LP task were: "B, E, H, L, N, P, S, V". For the MP task, children had to position: "February, April, July, September, November". The corresponding numerical lines were 1–21 for the Italian alphabet and 1–12 for the months of the year. These number lines started from 1 rather than 0 (also see Berteletti et al., 2010, who used 1–10 and 1–20 number lines) to match non-numerical lines that start with the first element of the sequence. Moreover, the numbers to be positioned corresponded to the serial position of the chosen non-numerical items (e.g., items "B", and "February" were replaced by number 2).

3. Results

3.1. Sequence knowledge

Mean correct responses and standard deviation were calculated for all sequences. The maximum score was 30 for numbers, 21 for letters (according to the Italian alphabet, all other letters were not considered) and 12 for months. Two scores were initially computed: one was the overall number of items produced for each sequence without repetitions and in any order; the second considered only items given in the correct order, without repetitions and with a maximum gap of 2 in-between items (e.g. "a, b, e, f, g..." was considered as an acceptable sequence but not "a, b, f, g..."). Correlations between the two types of scores were very high (from $r = .96$ to $r = .99$), therefore only the second scoring criterion was retained for the subsequent analyses. Means and *SDs* as a function of class and sequence are presented in Table 1. We also calculated scores according to the strictest criterion that allows no gaps. These are presented in Table 1 to allow comparison with the more lenient scoring criterion. We considered the latter as more representative of the child's sequence knowledge than the strictest criterion because some children (especially 3rd graders) skipped 1 or 2 items in an otherwise complete and correct sequence in the rush of showing how well they could perform (that was particularly evident for numbers; see Table 1). Note that the correlations between strict and lenient scores were very high (numbers: $r = .88$; letters: $r = .97$; months: $r = .97$) and that the results of all subsequent analyses involving sequence scores remained virtually unchanged when using the strict scores.

Separate one-way analyses of variance (ANOVA) on scores for each sequence were calculated introducing class as a factor. For all sequences, class was significant (Numbers: $F_{(3,\ 132)} = 20$, $p < .001$, $\eta^2 = .31$; Letters: $F_{(3,\ 131)} = 56$, $p < .001$, $\eta^2 = .56$; Months: $F_{(3,\ 131)} = 102$, $p < .001$, $\eta^2 = .70$). Post-hoc comparisons highlighted that the significant improvement, as could be expected, occurred between preschool and the primary school for all sequences (preschool vs. all primary grades for the three sequences: *ps* < .001, see Table 1). Moreover, between grade 1 and 3 a significant improvement also occurred for months ($p < .05$).

The a priori hypothesis was that improvement should occur with level of instruction for all sequences. All tasks positively correlated with class ($r = .48$, .69 and .77 for numbers, letters and months respectively) and with each other once class was partialled out (Numbers–Letters: $r = .42$; Numbers–Months: $r = .33$; Letters–Months: $r = .40$).

3.2. NP task

For the NP task, analyses were conducted according to the method recommended by Siegler and Booth (2004) and Siegler and Opfer (2003). Six children were excluded for not completing enough items on these lines. Estimation accuracy was computed using the Percentage of Absolute Error (PAE) of estimation for each participant (corrected with the formula $2^{*}\arcsin\sqrt{(PAE/100)}$ for statistical testing). This was calculated as follows:

Table 1
Mean Lenient and Strict scores (standard deviations in parenthesis) for the Sequence knowledge task.

Class	Numbers (Max score 30)			Letters (Max score 21)			Months (Max score 12)		
	M (*SD*) Lenient score	*M* (*SD*) Strict score	Range	*M* (*SD*) Lenient score	*M* (*SD*) Strict score	Range	*M* (*SD*) Lenient score	*M* (*SD*) Strict score	Range
Preschool ($n = 51$)	21.9 (8.9)	21.4 (9.2)	0–30	5.9 (6.9)[a]	5.1 (6.1)	0–21	1.9 (3.4)[a]	1.8 (3.1)	0–12
Grade 1 ($n = 28$)	29.4 (1.9)	29.4 (2)	20–30	16.8 (6.4)	15.6 (7.3)	0–21	9.4 (3.8)	8.9 (3.9)	1–12
Grade 2 ($n = 35$)	29.5 (1.9)	29.1 (3.4)	16–30	19.3 (4.6)	18.7 (5)	3–21	11.3 (2.1)	11 (2.6)	3–12
Grade 3 ($n = 22$)	30 (0)	28.6 (6.4)	0–30	20.3 (1.6)	19.5 (3.4)	8–21	11.9 (.5)	11.8 (.6)	10–12
Mean	26.7 (6.7)	26.2 (7.4)		13.9 (8.4)	13.1 (8.5)		7.5 (5.3)	7.2 (8.5)	

[a] One participant did not complete the task.

PAE = (estimate − real value)/line of estimates.

A mixed ANOVA on PAE was computed with class as a between-subject factor and interval (0–100 and 0–1000) as within-subject factor. Results indicated that both factors were significant (group: $F_{(3,\ 126)} = 38$, $p < .001$, $\eta^2 = .47$; interval: $F_{(1,\ 126)} = 355$, $p < .001$, $\eta^2 = .69$). Mean PAE was 17% (*SD* = 10%) for the 0–100 line and 32% (*SD* = 12%) for the 0–1000 line. Post-hoc comparison indicated that the precision of estimation significantly increased throughout classes (all *ps* < .005), except between grade 1 and 2. Mean PAEs across class (from preschool to grade 3) were 27%, 14%, 12% and 8% for the 0–100 line and 38%, 36%, 28%, and 19% for the 0–1000 line. These data closely resemble those obtained by Siegler and collaborators in their seminal studies (Booth & Siegler, 2006; Siegler & Booth, 2004). The interaction between class and interval was also significant, highlighting that the precision of estimation changed at different times for the two intervals ($F_{(3,\ 126)} = 10$, $p < .001$, $\eta^2 = .06$). Repeated contrasts indicated a significant improvement from preschool to grade 1 ($p < .001$) and from grade 2 to grade 3 ($p < .01$) for the 0–100 line. For the 0–1000 line, accuracy improved significantly from grade 1 to 2 ($p < .05$) and from grade 2 to 3 ($p < .001$).

To analyze the pattern of the estimates, the fit of logarithmic and linear functions were computed first on group medians and then for each individual child. For group medians, the difference between models was tested with a paired-sample *t*-test on the absolute distances between children's median estimate for each number and (a) the predicted values according to the best linear model and (b) the predicted values according to the best logarithmic model. If the *t*-test indicated a significant difference between the two distances, the best fitting model was attributed to the group. For preschoolers, both intervals were best represented by a logarithmic model (0–100 line: $t_{(9)} = 2.86$, $p < .05$, R^2-log = .93 and R^2-lin = .67; 0–1000 line: $t_{(9)} = 2.56$, $p < .05$, R^2-log = .83 and R^2-lin = .40). For grades 1 and 2, the estimates approached linearity for the 0–100 line but the two models were not statistically different (grade 1: R^2-lin = .88, R^2-log = .98; grade 2: R^2-lin = .95, R^2-log = .96). In contrast, their performance on the 0–1000 line was markedly logarithmic (grade 1: $t_{(9)} = 3.41$, $p < .01$, R^2-log = .93 and R^2-lin = .50; grade 2: $t_{(9)} = 3.72$, $p < .01$, R^2-log = .99 and R^2-lin = .69). The estimates of grade 3 children were best fit by the linear model for the smaller interval and approached linearity on the larger interval (0–100 line: $t_{(9)} = -3.22$, $p < .05$, R^2-lin = .99 and R^2-log = .89; 0–1000 line: R^2-lin = .85 and R^2-log = .94). Graphs of median estimates and the best fitting model are presented in Fig. 1.

Fitting individual children's estimates allows further characterization of the developmental patterns (Siegler & Opfer, 2003). The best fitting model between linear and logarithmic was attributed to each child whenever one was significant. If for example, both models were significant but the logarithmic R^2 was the highest then the child was attributed a logarithmic representation. In the case where both were not significant, the child was considered unable to position numbers. Therefore, for each line children could be classified as having linear, logarithmic or no representation.

Spearman's rank correlations were computed between class (preschool, grade 1, 2 and 3) and type of representation (no representation, logarithmic and linear) separately for each line (0–100 line: $r_s = .62$, $p < .001$, one-tailed; 0–1000 line: $r_s = .48$, $p < .001$, one-tailed). Moreover, the type of representation on one interval was significantly correlated with the type of representation on the other interval when class was partialled out ($r_s = .40$, $p < .001$, one-tailed). Table 2 reports the distribution of children (in percentages) as a function of type of representation

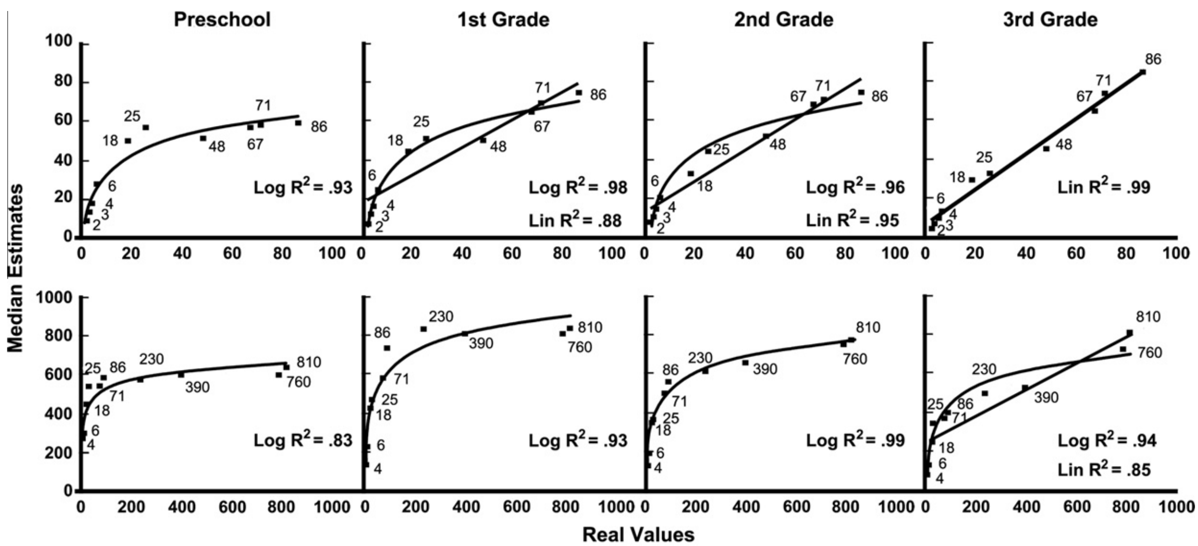

Fig. 1. Best logarithmic and linear models for each class in the Number-to-Position task. Median estimates are plotted against real values. Graphs for the 0–100 line are in the top row and those for the 0–1000 line are in the bottom row. Both models are plotted when they are not statistically different.

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Table 2
Distribution of children (%) as a function of the type of representation on the two number lines.

0–1000 Line	0–100 Line			Total
	None	Logarithmic	Linear	
None	16	4	1.5	21.5
Logarithmic	2	47	21	70
Linear	1.5	0	7	8.5
Total	19.5	51	29.5	100

Table 3
Distribution of children (%) as a function of the type of representation in the LP task and matched NP task.

LP task	1–21 NP task			Total
	None	Logarithmic	Linear	
None	15	6	5	26
Logarithmic	1	9	12	22
Linear	2	5	45	52
Total	18	20	62	100

on the two lines. Finally, correlations were also computed between type of representation for each line and counting score with class partialled out. The correlation was significant only for the 0–1000 line ($r_s = .27$, $p < .001$, one-tailed; $r_s = .09$, *n.s.* for the 0–100 line). The weak correlation is in agreement with the findings of Berteletti et al. (2010), who showed that the type of numerical representation in preschoolers had a good correlation with their scores in a numerical ordering task (either symbolic or non-symbolic) but it did not correlate with finger counting scores.

Overall, these results replicate and confirm previous findings on the NP task, that is, a developmental transition from a logarithmic to a linear representation (Berteletti et al., 2010; Siegler & Booth, 2004; Siegler & Opfer, 2003).

3.3. Non-numerical lines

Results for each non-numerical line and its matched number line are presented separately. All analyses followed the procedures used for the NP task. Six children had to be excluded from the analyses of the LP task and seven from the analyses of the MP task for not completing enough items and dropping out of the experimental session (5 children form preschool, 1 from grade 2 and 1 from grade 3).

3.3.1. LP task and 1–21 NP task

A mixed ANOVA on PAE was computed with class as a between-subject factor and type of line (alphabet line vs. 1–21 line) as a within-subject factor. The main effects of class and type of line were significant ($F_{(3,\ 126)} = 37$, $p < .001$, $\eta^2 = .47$, 25%, 11%, 11% and 10% from preschool to grade 3; $F_{(1,\ 126)} = 7.3$, $p < .01$, $\eta^2 = .05$), highlighting an improvement in estimation accuracy with education level and a better performance for the number line compared to the alphabet line. Mean PAEs were 17% ($SD = 12\%$) for the LP task and 15% ($SD = 11\%$) for the 1–21 NP task. A repeated contrast indicated a significant improvement in estimation accuracy from preschool to grade 1 ($p < .001$).

For group medians, preschoolers' estimates in the LP task were best represented by a logarithmic function (R^2-log = .84 and R^2-lin = .66, $t_{(7)} = 2.79$, $p < .05$) whereas for the corresponding NP task the difference between the two types of representations was not significant (R^2-lin = .95 and R^2-log = .98). The good fit of the linear model indicates that preschoolers approach linearity on a small numerical interval (Berteletti et al., 2010). In grades 1, 2 and 3, the way children positioned letters was equally well fit by the linear and the logarithmic models (R^2-lin = .98, .97, .99, respectively; R^2-log = .92, .87, .91, respectively), whereas numbers were positioned linearly (grade 1: R^2-lin = .99 and R^2-log = .89, $t_{(7)} = -2.82$, $p < .05$; grade 2: R^2-lin = .98 and R^2-log = .83, $t_{(7)} = -2.77$, $p < .05$; grade 3: R^2-lin = .97 and R^2-log = .84, $t_{(7)} = -3.21$, $p < .05$; Fig. 2).

Subsequently, for each task (LP and 1–21 NP) each child was assigned a linear, logarithmic or no representation. One-tailed Spearman's rank correlations were significant between class and type of representation, separately for each line (LP: $r_s = .53$, $p < .001$; 1–21 NP: $r_s = .57$, $p < .001$) as well as between the representations for the two lines (Table 3) once class was partialled out ($r_s = .47$, $p < .001$).

3.3.2. MP task and 1–12 NP task

The mixed ANOVA on PAE showed a significant main effect of class ($F_{(3,\ 125)} = 25$, $p < .001$, $\eta^2 = .38$; mean PAEs 26%, 14%, 14% and 10% from preschool to grade 3), a main effect of tasks ($F_{(1,\ 125)} = 5.58$, $p < .05$, $\eta^2 = .04$; mean PAEs 17%, 18%, respectively for the MP task and the NP task) and a significant interaction between class and task ($F_{(3,\ 125)} = 5.35$, $p < .005$, $\eta^2 = .11$). Preschoolers were more

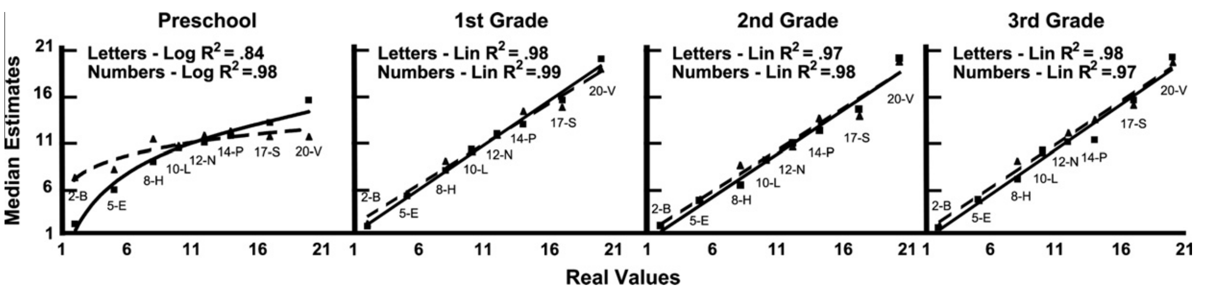

Fig. 2. Best fitting models for each class in the LP task and the matched 1–21 NP task. Median estimates are plotted against real values for numbers (dots) and the ordinal value for letters (triangles). The models' fits are represented with dashed lines for the LP task and solid lines for the NP task.

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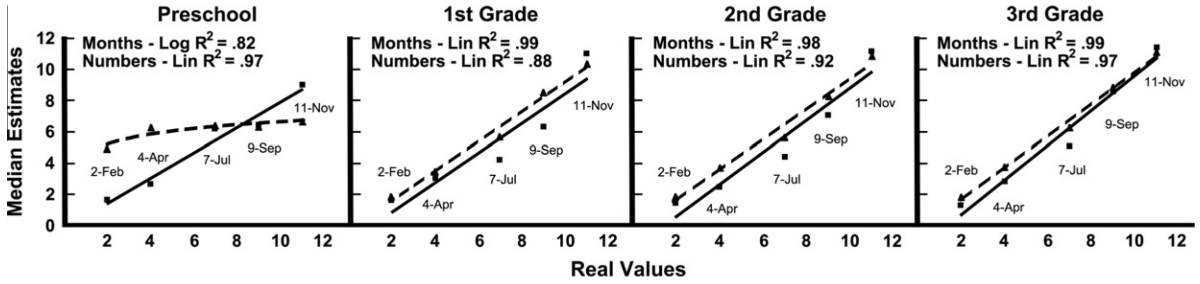

Fig. 3. Best fitting models for each class in the MP task and the matched 1–12 NP task. Median estimates are plotted against real values for numbers (dots) and the ordinal value for months (triangles). The models' fits are represented with dashed lines for the MP task and solid lines for the NP task.

accurate with numbers than months (mean PAE 24% vs. 29%), though the comparison did not reach significance ($F_{(1,\ 45)} = 3.7$, $p = .06$, $\eta^2 = .07$). In contrast, grade 1 and grade 3 children were more accurate in positioning months than numbers (grade 1: mean PAE 12% vs. 16%, $F_{(1,\ 26)} = 6.69$, $p < .05$, $\eta^2 = .20$; grade 3: mean PAE 8% vs. 13%, $F_{(1,\ 20)} = 11$, $p < .005$, $\eta^2 = .36$).

Subsequently, the fit of logarithmic and linear functions were computed on group medians first and then for each individual child. The *t*-test on the logarithmic and linear models for preschoolers' performance in both MP and matched NP tasks did not reach significance, indicating that both models equally fit the data (Fig. 3). Nevertheless, a slight better R^2 was found for the logarithmic fit when positioning months and a better linear R^2 was found for the matched number line (R^2-log = .84 and R^2-lin = .68 for the MP task, R^2-log = .90 and R^2-lin = .97 for the 1–12 NP task). First graders positioned months linearly ($t_{(4)} = -3.74$, $p < .05$, R^2-lin = .99 and R^2-log = .90), whereas the fits of the two models did not differ for the numerical estimates (R^2-log = .74 and R^2-lin = .88). For the two older groups, both tasks were best fit by a linear model (MP task: $t_{(4)} = -3.48$, $p < .05$, R^2-lin = .98 and R^2-log = .89 for grade 2 and $t_{(4)} = -3.7$, $p < .05$, R^2-lin = .99 and R^2-log = .92 for grade 3; the 1–12 NP task, $t_{(4)} = -2.85$, $p < .05$, R^2-lin = .92 and R^2-log ^{2} = .78 for grade 2 and $t_{(4)} = -3.7$, $p < .05$, R^2-lin = .97 and R^2-log = .86 for grade 3).

Again, each child was attributed a representation for the MP and the 1–12 NP tasks. Therefore, for each line, children could have a linear, logarithmic or no representation. One-tailed Spearman's rank correlations were significant between class and type of representation for each task (MP task: $r_s = .59$, $p < .001$; 1–12 NP task: $r_s = .39$, $p < .001$) as well as between the representations for each task (Table 4) with class partialled out ($r_s = .33$, $p < .001$).

Table 4
Distribution of children (%) as a function of the type of representation in the MP task and matched NP task.

MP task	1–12 NP task			Total
	None	Logarithmic	Linear	
None	22	2	13	37
Logarithmic	3	1	9	13
Linear	5	5	40	50
Total	30	8	62	100

3.4. Regression analyses

The analyses of the LP and MP tasks showed that children progress to a linear positioning of the items, even though this is not explicitly required by the task and it is not formally implied by the knowledge of the non-numerical order. For example, knowledge of the alphabet does not imply that the distance between "A" and "C" is twice the distance between "A" and "B". There are two possible explanations for the linearization of the representation of non-numerical sequences. One is that children generalize the linearity principle to all ordinal sequences, whether or not they are numerical, as long as the context is familiar. The alternative hypothesis is that logarithmic and linear positioning do not depend on the properties of the numerical domain but represent a general regularization phenomenon: that is, the more items of the sequence are known, the more they are regularly spaced onto the line. To investigate this issue, we performed a set of fixed-entry multiple regression analyses to account for the type of representation deployed by individual children in the various estimation tasks. Class was always introduced as first predictor to partial out the general effect of schooling. The score on the relevant sequence (numbers, alphabet or months) was introduced as second predictor to assess the role of specific sequence knowledge. Indeed, as shown by Siegler and colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003 also see Berteletti et al., 2010), children's ability to position numbers in the NP task is strongly influenced by the familiarity with the numerical context, that is, how well children master the numbers included in the interval. Finally, for the LP and MP tasks, we introduced as third predictor the type of representation displayed by the child on the matched NP task (1 for no representation, 2 for logarithmic and 3 for linear).

If the latter explains unique variance, we can conclude that the representation deployed in the non-numerical task is influenced by the child's numerical representation and it cannot be fully explained by specific knowledge of the sequence. Table 5 presents the percentages of unique variance explained by each predictor in the regression analyses.

Inspection of Table 5 reveals that even though the largest proportion of variance was explained by educational level, domain-specific knowledge (i.e., score on the relevant sequence) significantly influenced the ability to estimate in a linear way across all tasks, whether numerical or

Table 5
Variance explained (%) at each step of the fixed-entry multiple regression analyses.

1st Predictor					
(N participants)	LP *(129)*	MP *(127)*	1–21*(132)*	1–12*(133)*	
	Class	27**	33**	32**	15**
2nd Predictor					
	Alphabet score	10**			
	Month score		10**		
	Number score			10**	6**
	Total variance	37	43	42	22
3rd Predictor					
	Representation 1–21	11**			
	Representation 1–12		3*		
	Total variance	48	46		

Note: Each column corresponds to the percentage of unique variance explained by each predictor. Class was introduced as first predictor, score on the relevant sequence as second predictor and, for the LP and MP tasks, type of representation for the matched NP task.
* $p < .05$.
** $p < .001$.

non-numerical. More crucially, the representation deployed in the non-numerical tasks was significantly influenced by the quality of numerical representation even after partialling out the effects of schooling and domain-specific knowledge.

To further investigate the role of sequence knowledge in achieving linear positioning of numbers, letters and months, we selected children who obtained 100% correct in the sequence knowledge task. The hypothesis that linearity of estimation is a general regularization phenomenon leads to the prediction that these children would only deploy linear positioning. For numbers, we focused on the 1–21 NP task because sequence knowledge for numbers was only assessed up to 30. Ninety-three children had ceiling scores in the counting task: 73% of them were linear, whereas 23% were logarithmic and 4% were unable to position items in a meaningful way. For the LP task, of the 63 children who were at ceiling in the alphabet score, 73% showed linear positioning, 21% showed logarithmic positioning and the remaining 6% were unable to position items in a meaningful way. For the MP task, of the 67 children who were at ceiling in the months score, 76% positioned months linearly, 15% showed logarithmic positioning and the remaining 9% were unable to position items in a meaningful way. These results show that perfect sequence knowledge is not a sufficient condition for linearity of estimation.

Finally, to further support our claim that linearity in the numerical domain precedes linearity in the non-numerical domain, we assessed whether the type of representation in the numerical domain, across the entire sample of children, had a higher rank (1 = no representation, 2 = logarithmic and 3 = linear) than the representation in the non-numerical domain. A one-tailed Wilcoxon matched pair test for ordinal-categorical variables showed that this was true for both LP and MP tasks (1–21 NP vs. LP: $Z = 3.133$, $p < .001$, $r = .276$; 1–12 NP vs. MP: $Z = 2.124$, $p < .05$, $r = .187$).

4. Discussion

The aim of the study was to investigate whether children can map non-numerical ordered sequences onto lines and whether the type of representation deployed reveals a similar developmental trend as it has been shown for numbers (Siegler & Opfer, 2003; Siegler & Booth, 2004; Berteletti et al., 2010).

First, we replicated previous results on 0–100 and 0–1000 number lines. Children showed the classical developmental pattern with an initial logarithmic phase followed by a shift to linear positioning. Moreover, some children showed different patterns according to the interval: they positioned numbers linearly on the most familiar interval but reverted to a logarithmic positioning on the larger and less familiar interval.

Second, we compared performance on non-numerical lines (i.e. LP and MP tasks) with their matched numerical lines (i.e. 1–21 and 1–12 NP tasks). In all tasks we observed an increase of accuracy in estimating the correct position of items with age group and the largest improvement occurred from preschool to grade 1. But most interestingly, we observed for the first time a developmental trend for the two non-numerical tasks, similar to the one observed for the NP tasks. Indeed, analyses on group and single-subject estimates have shown for the LP and MP tasks that there seems to be a mandatory logarithmic transition preceding linearity. This result supports the hypothesis of a common developmental pattern for numbers and non-numerical ordered sequences.

To further understand whether the time of the developmental shifts are independent across domains or whether the linear positioning of non-numerical ordered sequences is observed only once it is acquired for a comparable numerical range, we computed a set of fixed-entry regressions for the LP and MP tasks. These analyses revealed that the type of representation deployed in the matched NP task (i.e., 1–21 or 1–12) accounted for unique variance in the non-numerical tasks even after partialling out the effects of schooling and domain-specific knowledge. That is, the quality of the representation deployed (i.e., none, logarithmic or linear) in the numerically matched interval was predictive of how items were positioned on the non-numerical interval. This result supports the hypothesis that linearity of non-numerical ordered sequences is a generalization from the numerical domain. Indeed, the type of representation in numerical estimation had higher rank than the type of representation in both non-numerical estimation tasks. Finally, it is important to stress that knowledge of the alphabet does not necessarily imply a representation in which adjacent letters have fixed, equal distance between each other. Indeed, performance of the youngest children in the LP and MP tasks did not conform to a linear representation. Crucially, even perfect knowledge of the sequences was not sufficient to yield linearity. Though the majority of children who scored at ceiling in sequence knowledge had acquired linearity, one child out of four either positioned items logarithmically or failed to show a meaningful mapping (27% for the 1–21 NP and LP tasks, and 24% for the MP task).

It is worth noting at this point that the interpretation of the shift from logarithmic to linear positioning has been recently challenged by Barth and Paladino (2011). They argued that performance in the NP task is better captured by a model of proportion judgment (which implies a sigmoidal rather than logarithmic function) and that the apparent logarithmic compression is an artifact of under-sampling the larger values in the range. Regardless of the outcome of the debate raised by these claims (see Opfer, Siegler, & Young, 2011, for a convincing rebuttal), two aspects of our data deserve discussion. First, the sampling of items in the non-numerical positioning tasks and the numerically matched number lines was evenly distributed across the respective ranges, yet no sigmoidal distribution of estimates was visible (Figs. 2 and 3). Second, and most important, our finding that linearity is generalized from numerical to non-numerical domains does not depend on the interpretation of the observed mapping behavior as logarithmic or sigmoidal.

Taken together, our results exclude the possibility that linear positioning is a general regularization phenomenon but support the generalization hypothesis, whereby linearity is a distinctive feature of numbers, perhaps driven by their unique property of conveying cardinality. This interpretation would account for the common effects observed across domains. Indeed, if linearity is generalized to all ordinal sequences, it is conceivable that non-numerical sequences might acquire other characteristics that were initially thought to be distinctive of numbers. This could explain the distance effect (Jou & Aldridge, 1999) and the SNARC-like effect observed for non-numerical sequences (Gevers et al., 2003, 2004). The acquisition of common characteristics might lead to activation of the same cortical regions, including the hIPS (Fias et al., 2007; Ischebeck et al., 2008; but see Zorzi et al., 2011), during processing of both numerical and non-numerical order.

Note, however, that the generalization hypothesis does not imply that the format of representation for non-numerical ordered sequences is identical to that of numbers. For example, the qualitative difference between representations is highlighted by the dissociation between numbers and letters found by Van Opstal and collaborators (2008) in a priming task. Indeed, only number priming revealed an overlapping distribution of activations on the internal representation. Overlapping representations between adjacent letters would be clearly dysfunctional because the conventional ordering of letters in the alphabet is arbitrary and it bears no significance for written language processing. In the same vein, the spatial coding of letters and months does not need to be identical to that of numbers (Zorzi et al., 2006). For all these reasons, patterns of brain activity during comparison of non-numerical order can be distinguished from number comparison even though they recruit partially overlapping regions, such as the hIPS (Zorzi et al., 2011). Van Opstal, Verguts, Orban, and Fias (2008) found that the consolidation of new ordinal sequences is correlated with activation of the angular gyrus, which is known to be involved in the verbal coding of numbers and in arithmetic facts retrieval (Dehaene et al., 2003). They suggested that the hIPS would be recruited only after extensive practice with a new ordinal sequence. However, 1 week of practice still failed to activate hIPS in a follow-up study (Van Opstal, Fias, Peigneux, & Verguts, 2009). This suggests that the involvement of IPS requires a higher level of abstraction of the order implied by a new sequence, perhaps mediated by links to the number domain.

In summary, the present study shows that the developmental transition from logarithmic to linear positioning of items on a spatial scale is not a distinctive feature of numbers but is shared by non-numerical ordered sequences. Nonetheless, the quality of numerical representation is predictive of the pattern shown by children in positioning non-numerical items, whereas perfect knowledge of the ordered sequence is not enough to yield linearity. We therefore conclude that the principle of linearity is generalized from the numerical domain to non-numerical ordered sequences.

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